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North Jordan
RIVER NENE
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Regten Row
TOWN OF OUNDLE
BACK LANE
OUNDLE FIELD
Scale of Chains.
1 2 3 4 5 6 7 8

Scale of Chains.

1 2 3 4 5 6 7 8 9

A

TREATISE

ON

LAND-SURVEYING,

IN SEVEN PARTS.

PART I.

Contains Definitions and Problems in Geometry.

PART II.

Rules for finding the Areas of Plane Figures.

PART III.

To Survey with the Chain and Cross.

PART IV.

To Survey with the Chain only.

PART V.

Rules for laying out and dividing Land.

PART VI.

A Full Explanation of the Method used by the most eminent Surveyors, in measuring and planning a Farm or a Lordship with the Chain only; with Plates exhibiting the progressive Steps of planning a small Farm.

PART VII.

To Survey by measuring the Angles and Lines.

THE WHOLE ILLUSTRATED WITH

NEARLY TWO HUNDRED DIAGRAMS, AND A COLOURED PLAN OF AN ESTATE.

BY THOMAS DIX.

SEVENTH EDITION, CORRECTED AND GREATLY IMPROVED

BY SAMUEL MAYNARD.

LONDON:

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1841.

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ST. JOHN'S SQUARE.

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EDITOR'S PREFACE.

It has been the object of the Editor of this *Sixth Edition* to make such alterations and improvements, without materially departing from the original plan of the work, as he trusts will render it still more useful to Students and Practical Surveyors.

The principal alterations are to be found in the first, second, and fifth parts, of which the following is a brief enumeration.

In the first part several useless problems have been suppressed, and their places filled up with others of greater importance.—The second part has been greatly extended, by additional problems, notes, and examples.—The fifth part has been completely re-modelled, and will be found to contain the most useful and essential problems which are to be met with in actual practice. The whole of the book has been carefully revised, and every example re-worked, which induces the Editor to speak with confidence

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of the general accuracy of the work. Perspicuity has been kept constantly in view; being intended for those who possess only a knowledge of Arithmetic, it became necessary to make it as clear and intelligible as possible. How far the Editor may have succeeded in his endeavours to lessen the burden of the student, or what merit may be due to his labours, must be left to the judgment of an impartial public.

SAMUEL MAYNARD.

London,
October 20th, 1834.

ADVERTISEMENT

TO THE

SEVENTH EDITION.

The present Edition has been revised with considerable care. In Part the Second three additional problems have been introduced, and a more extensive view of the tables relating to regular polygons up to fifty sides, which the Editor has carefully recomputed for this Edition; also, a great variety of questions for exercise has been added at the end. Beside these additions, notes and illustrative remarks (wherever any obscurity was likely to be felt by the learner) have been introduced in various parts of the work. The Editor, therefore, submits this Edition to the judgment of experienced teachers and practical surveyors; and should he be successful in securing their approbation, his expectations will be fully realized.

SAMUEL MAYNARD.

No. 8, Earl's Court, Leicester Square,
London, March 20th, 1841.

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EXPLANATION OF THE CHARACTERS
USED IN THIS WORK.

$+$ denotes *plus*, or *more*. The sign of *addition*, signifying that the numbers between which it is placed are to be added together. Thus, $9+6$ denotes that 6 is to be added to 9. In geometrical lines also, A B$+$C D signifies that the line A B is to be added to the line C D.

$-$ denotes *minus*, or *less*. The sign of *subtraction*, signifying that the latter of the two numbers between which it is placed is to be taken from the former. Thus, $5-3$ denotes that the 3 is to be taken from the 5. In geometrical lines also, A B$-$C D signifies that the line C D is to be subtracted from the line A B.

$\wp$ denotes *difference;* as $a \wp b$ shows that the difference between a and b is to be taken, when it is not known which is the greater.

$\times$ *or* . denotes *into*, or *by*. The sign of *multiplication*, signifying that the numbers between which it is placed are to be multiplied together. Thus 8×6 denotes that 8 is to be multiplied by 6. In geometrical lines also, A B$\times$C D signifies that the line A B is to be multiplied by the line C D. Again $a+b$ multiplied by $a-b$ is sometimes expressed thus $(a+b) \cdot (a-b)$.

$\div$ denotes *divided by*. The sign of *division*, signifying that the former of the two numbers between which it is placed is to be divided by the latter. Thus, $8\div 4$ denotes that 8 is to be divided by 4. This is also expressed by placing the dividend above a line, and the divisor below it. Thus, $\frac{8}{4}$ *numerator* / *denominator* denotes that 8 is to be divided by 4. In geometrical lines also, A B$\div$C D signifies that the line A B is to be divided by the line C D; or thus, $\frac{\text{A B}}{\text{C D}}$

$\because$ is to
$::$ as
$\therefore$ is to

denote *proportionals*, signifying that the numbers between which they are placed are *proportional*. Thus, $2 \because 4 :: 8 \because 16$, denotes that the number 2 bears the same proportion to 4 as 8 does to 16, and is usually read 2 is to 4, as 8 is to 16.

$=$ denotes *equal to*. The sign of equality, signifying that the numbers between which it is placed are equal to each other. Thus, 2 poles$+$2 poles$=$4 poles$=$22 yards$=$1 chain$=$100 links.

$\overline{\quad}$ denotes a *vinculum*; and ( ) denotes a *parenthesis*; [ ] denotes a *crotchet*; and $\{\}$ denotes a *brace*. These signs are made use of to connect two or more quantities together, and they are synonymous with regard to their application; for,

$$\overline{(7+4-5)}\times 8=[(7+4)-5]\times 8=(11-5)\times 8=6\times 8=48,$$

is the same as

$$\{(7+4)-5\}\times 8=(11-5)\times 8=6\times 8=48.$$

The *vinculum*, or *parenthesis*, which includes the 7 and 4, serves as a chain to link them together, and shows that they are to be added together before the number 5 is subtracted; and the *crotchet*, and also the *brace*, shows that the numerals which it includes must be operated upon, and the result multiplied by the number 8.

^{2} ^{3} ^{n} denote *indices* or *exponents*. These expressions, placed above a quantity to the right, show to what power it is understood to be raised; as, 10^2, 10^3, a^n, &c., which represents the square and cube of 10; and also the nth power of a; thus, $10^2=10\times 10=100$, and $10^3=10\times 10\times 10=1000$; also a^n, meaning that a is a factor as often as there are unities in n, whatever number n may be.

$\surd$, $\sqrt[3]{}$, $\sqrt[n]{}$ denotes *radical signs*. Any one of these expressions, placed before a quantity, represents that the quantity is to have its respective root extracted; it is also expressed by a fractional index, or exponent, placed over, and a little to the right of the given quantity. Thus, the square root of 36 is expressed by $\surd 36$, or $(36)^{\frac{1}{2}}$; the cube root of 64 by $\sqrt[3]{64}$, or $(64)^{\frac{1}{3}}$; and the nth root of a by $\sqrt[n]{a}$, or $a^{\frac{1}{n}}$, &c.

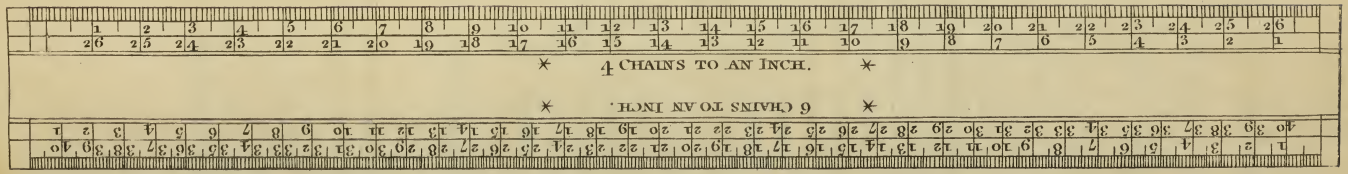
$\angle$ denotes an *angle*; as, $\angle$A, signifies the angle A.

$\therefore$ denotes *therefore*.

$\because$ denotes *because*.

$^{\circ}$ $^{\prime}$ $^{\prime\prime}$ denote *degrees*, *minutes*, and *seconds*, when placed above a quantity to the right; thus, 70 degrees, 16 minutes, 34 seconds, are expressed by 70°, $16^{\prime}$, $34^{\prime\prime}$. Also, one of these characters is made use of to denote a *repeating decimal*, when placed above the quantity to the right; thus, the *repeater* ·819444, &c. is expressed by ·8194′; and the same dash denotes a *circulate*, when placed over its first and last figure to the right; thus, ·769230769230, &c. is expressed by ·7′69230′

PLOTTING SCALE.



The use of this Scale is too obvious to need much explanation. Every long division marked 1.2.3 &c is 1 Chain or 100 Links & every short division 10 Links.

London Published April 1840 by Whittaker & Co Ave Maria Lane.

PRACTICAL GEOMETRY.

PART THE FIRST.

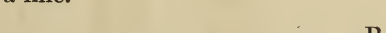
GEOMETRY is a science which treats of the descriptions, properties, and relations of magnitudes or extension; as solids, surfaces, lines, and angles.

DEFINITIONS.

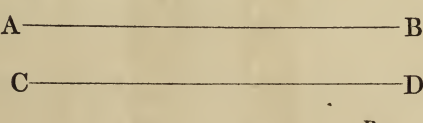
1. A *point* has position, but not magnitude.

2. A *line* is length without breadth, and its bounds or extremes are points.

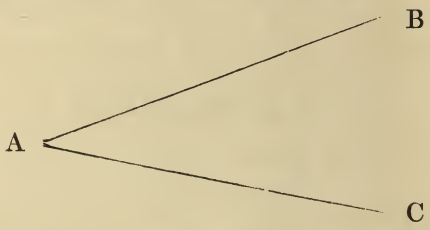
3. A *straight line* never changes its direction, and is the *shortest* distance between two points, thus A B represents a line.



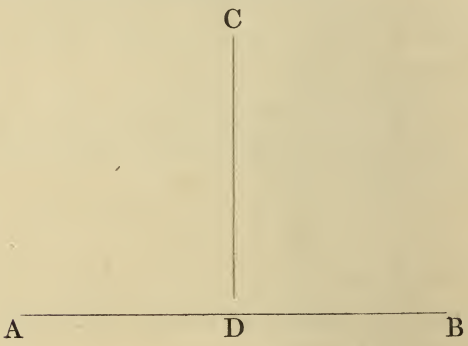
4. *Parallel lines* are always at the same distance; and never meet, though ever so far produced; thus C D is parallel to A B.



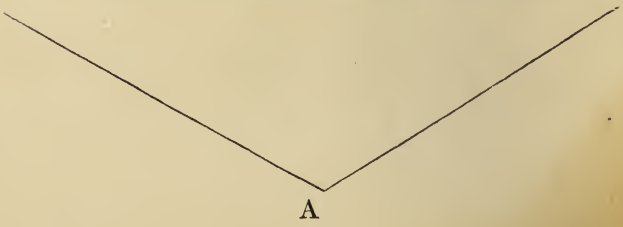
5. An *angle* is the opening or inclination of two lines, which meet: the point of meeting is called the *vertex*, as A; and the lines which are said to contain the angle are called its *sides*, as A B, A C.



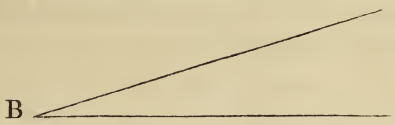
6. When one straight line C D, stands on another straight line A B, and makes the adjacent angles A D C and B D C on each side equal, both angles are *right* angles, and the line C D, which stands on the line A B, is *perpendicular* to it.



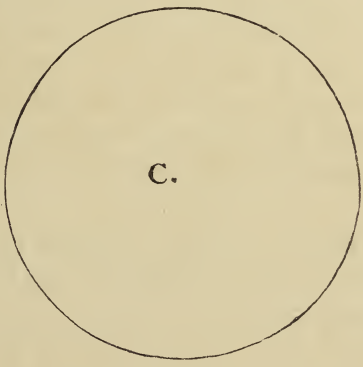
7. An *obtuse* angle is greater than a right angle, as A.



8. An *acute* angle is less than a right angle, as B.



9. A *circle*[1] is a plane figure bounded by a curved line, called the *circumference*, which is every where equally distant from a point C within the circle, called the *centre*.

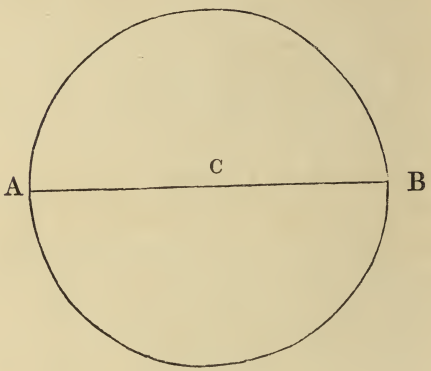


[1] There is no figure that affords a greater variety of useful properties than the circle. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter, or has the least perimeter about the same area.

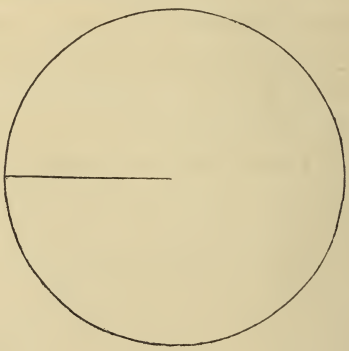
The area of a circle is always less than the area of any regular polygon circumscribed about it, and its circumference always less than the circumference of the polygon. But, on the other hand, its area is always greater than that of its inscribed polygon, and its circumference greater than the circumference of its inscribed polygon; hence the circle is always limited between these polygons. The area of a circle is equal to that of a triangle, whose base is equal to the circumference, and perpendicular equal to the radius.

Circles, like other similar plane figures, have the same ratio to each other, as the squares of their diameters: and the circumferences of circles, are to one another as their diameters, or radii.—*Keith's Geometry, by S. Maynard*, book vii. prop. 195 and 196, &c.

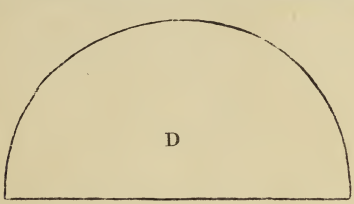
10. The *diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the circumference, as A B.



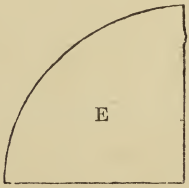
11. The *radius* of a circle is a straight line drawn from the centre to the circumference, and is equal to half the diameter, as A C.



12. A *semicircle* is half a circle, and is contained by half the circumference and a diameter, as D.



13. A *quadrant* is a quarter of a circle, and is contained by two radii and a quarter of the circumference, as E.

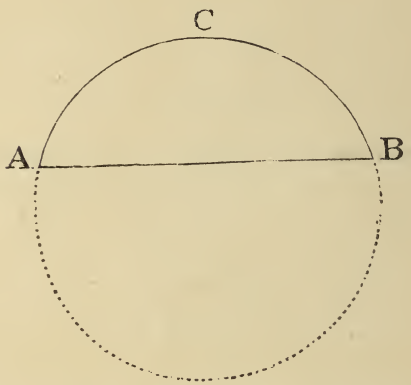


14. (*See the diagram at Def.* 16.) A straight line cutting off any portion of a circle, greater or less than a semicircle, and bounded by the circumference, is called a *chord;* as A B.

15. And the part of the circumference which is cut off by A B is called an *arc;* as A C B.

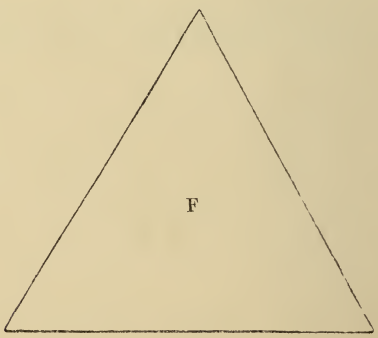
16. A *segment* of a circle is a space included by an arc and its chord.

Thus the space A B C A, included by the arc A C B, and the chord A B, is a segment.

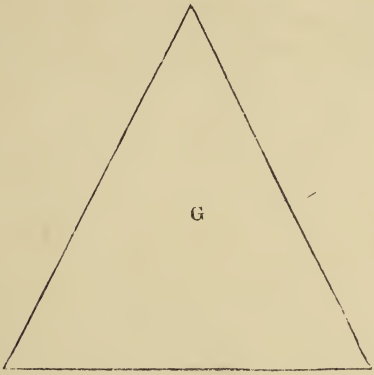


17. All plane figures, bounded by three right lines, are called *triangles*, and receive different denominations, according to the nature of their sides and angles.

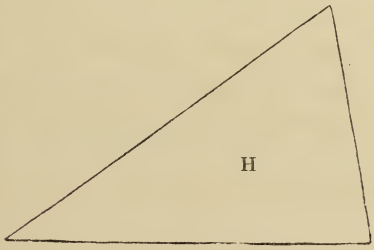
18. An *equilateral* triangle has all its sides equal, as F.



19. *An isosceles* triangle has two of its sides equal, as G.

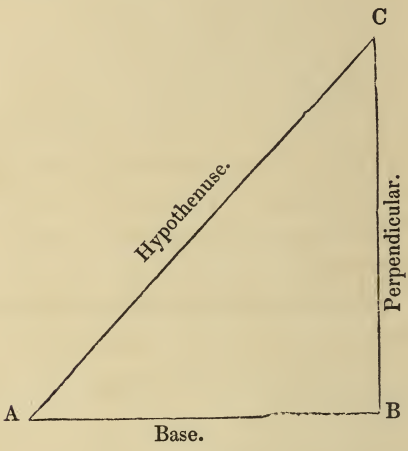


20. A *scalene* triangle has all its sides unequal, as H.

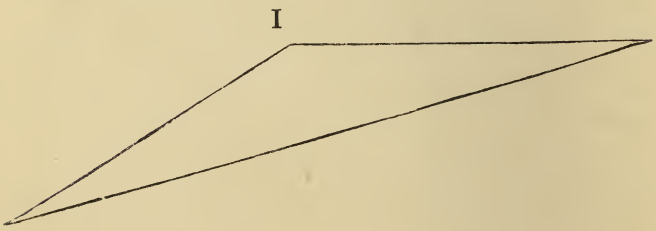


21. A *right-angled* triangle is that which has one right angle; as A B C.

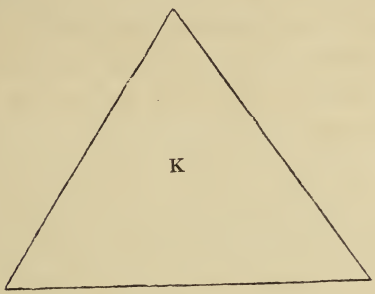
The side A C, opposite to the right angle, is called the *hypothenuse;* the side B C, the *perpendicular;* and the side A B, on which the triangle stands, is called the *base.*



22. An *obtuse-angled* triangle has one obtuse angle in it, as I.



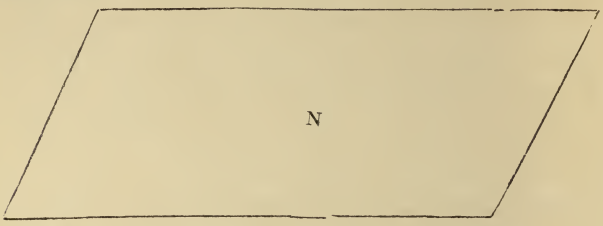
23. An *acute-angled* triangle has all its angles acute, as K.



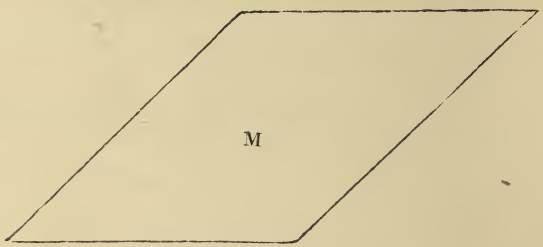
24. All plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals;* and receive different names according to the nature of their sides and angles.

25. A *square* is a quadrilateral, having all its sides equal and all its angles right angles, as L.

26. A *parallelogram* has its opposite sides parallel and equal, as N.

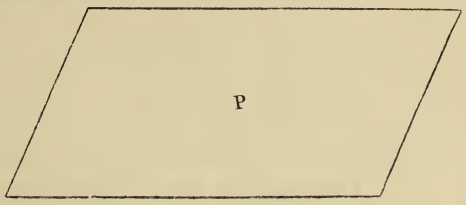


27. A *rhombus* is a quadrilateral parallelogram, having all its sides and opposite angles equal, but its angles are not right angles, as M.



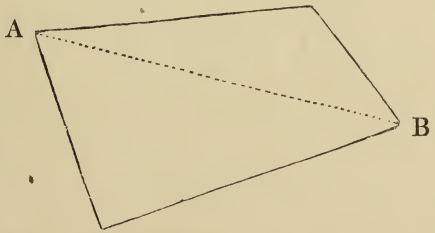
28. A *rectangle* is a parallelogram having its opposite sides equal and all its angles are right angles, as O.

29. A *rhomboid* is a parallelogram, having its opposite sides and angles equal, but its angles are not right angles, as P.

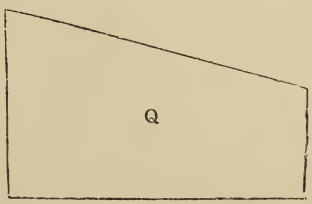


30. A *trapezium* is a quadrilateral, having no two of its sides parallel.

A straight line connecting two opposite angles of the trapezium, as A B, is called a diagonal.



31. A *trapezoid* is a quadrilateral, having two of its opposite sides parallel, and the remaining two not parallel, as Q.



32. *Plane* figures, having more than four sides, are in general called *polygons;* and they receive particular names, according to the number of their sides and angles.

33. A *pentagon* is a polygon of 5 sides; a *hexagon,* 6; a *heptagon,* 7; an *octagon,* 8; a *nonagon,* 9; a *decagon,* 10; an *undecagon,* 11; a *duodecagon,* 12; a *tridecagon,* 13; a *tetradecagon,* 14; a *pentadecagon,* 15; a *hexadecagon,* 16; a *heptadecagon,* 17; an *octadecagon,* 18; a *nonadecagon,* 19; an *eicosagon,* 20; an *uneicosagon,* 21; a *duoeicosagon,* 22; a *triseicosagon,* 23; a *tetreicosagon,* 24; a *penteicosagon,* 25; a *hexeicosagon,* 26; a *hepteicosagon,* 27; an *octeicosagon,* 28; a *noneicosagon,* 29; a *triacontagon,* 30; an *untriacontagon,* 31; a *duotriacontagon,* 32; a *tritriacontagon,* 33; a *tetratriacontagon,* 34; a *pentriacontagon,* 35; a *hextriacontagon,* 36; a *heptriacontagon,* 37; an *octriacontagon,* 38; a *nonatriacontagon,* 39; a *tesseracontagon,* 40; an *untesseracontagon,* 41; a *duotesseracontagon,* 42; a *tritesseracontagon,* 43; a *tetratesseracontagon,* 44; a *pentesseracontagon,* 45; a *hextesseracontagon,* 46; a *heptesseracontagon,* 47; an *octesseracontagon,* 48; a *nonatesseracontagon,* 49; a *pentecontagon,* 50 sides.

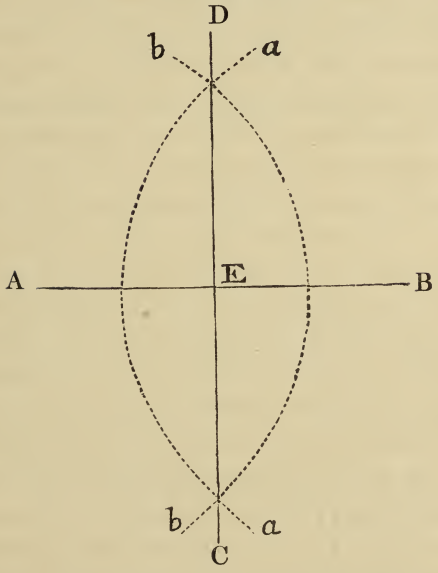
34. The polygon is regular when it has all its sides, as well as its angles, equal. If the sides or angles are unequal, the polygon is said to be irregular.

35. An equilateral triangle is also a regular figure of three sides, called a *trigon;* and a square of four, denominated a *tetragon.*

PROBLEMS.

PROBLEM I.

To bisect a given Line A B.



On the centre B with any radius greater than half the line, describe the arc *a a*.

With the same radius, and from A as a centre, describe the arc *b b*.

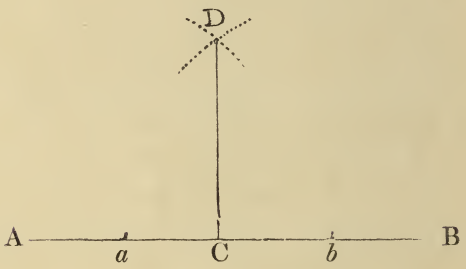
Through the points of intersection draw the line C D, which will divide the given line into two equal parts.

Note.—E D is a perpendicular raised in the middle of the line A B.

PROBLEM II.

To erect a Perpendicular from a given Point C *in a given right Line* A B.

When the given point is near the middle of the line.



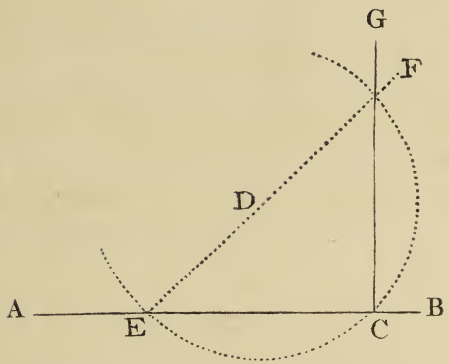
Set off two equal distances from it on the line A B, as C *a* and C *b*.

With any radius greater than *a* C, and from *a* as a centre, describe an arc.

With the same radius, and from *b* as a centre, describe another arc, cutting the former in D.

Draw the line C D, which will be the perpendicular required.

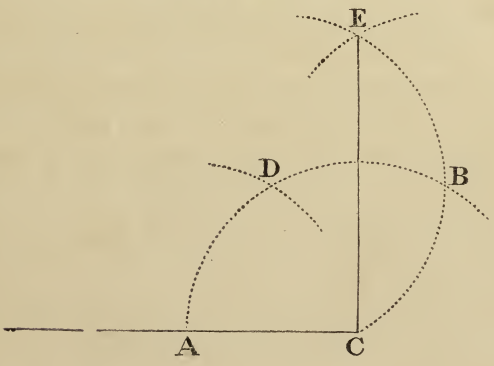
When the given point C *is at or near the end of the line.*



Take any point, as D, for a centre, and with the radius D C, describe an arc, cutting A B in E and C. Draw the line E F through the point D, cutting the arc in F.

Through the intersection F draw the line C G, which will be the perpendicular required.

SECOND METHOD.



With the given point C as a centre, and any convenient radius, describe the arc A B.

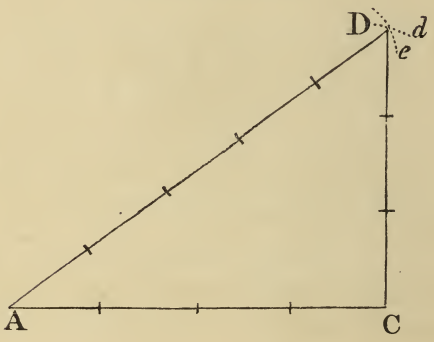
With the same radius, and from A as a centre, cross the arc in D.

With the same radius, and from D as a centre, describe the arc E B C, cutting the arc A D B in B.

With the same radius, and from B as a centre, cross the last arc in E.

Draw the line E C, which will be the perpendicular required.

THIRD METHOD.



From a scale of equal parts, set off four on the given line from C towards A.

Take three parts for a radius, and from the centre C describe the arc *d*.

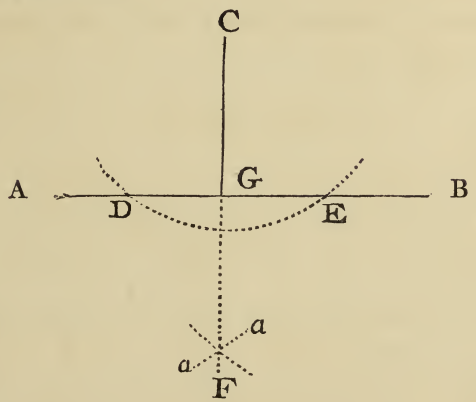
Take five parts for a radius, and from the centre A describe the arc *e*.

Through the intersection D draw the line D C, which will be the perpendicular required.

PROBLEM III.

To let fall a Perpendicular from a given Point C, *on a given right line* A B.

CASE I.—*When the point is nearly opposite the middle of the line.*



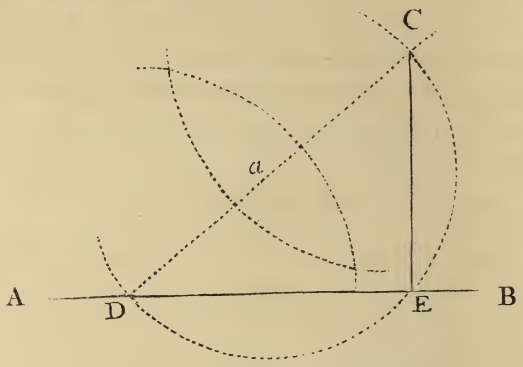
From C as a centre, describe an arc cutting the given line in D and E.

From D as a centre, and a radius longer than D G, describe the arc *a a*.

With the same radius, and from E as a centre, describe the arc cutting the first in F.

Draw the line C F.—C G is the perpendicular required.

CASE II.—*When the given point* C *is nearly opposite the end of the line.*



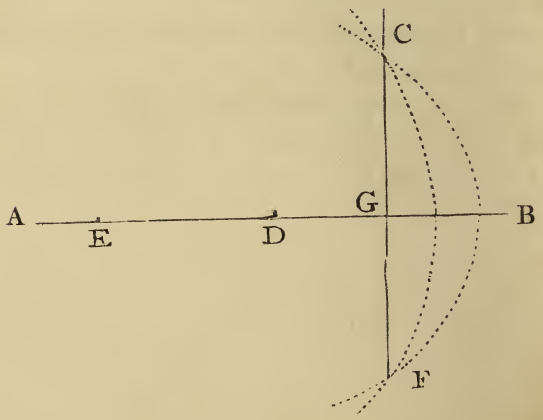
From the point C, draw a line meeting the given line in D.

Bisect C D in *a*.

With the radius *a* C, and *a* as a centre, describe the arc C E D.

Draw the line C E, which will be the perpendicular required.

ANOTHER METHOD.



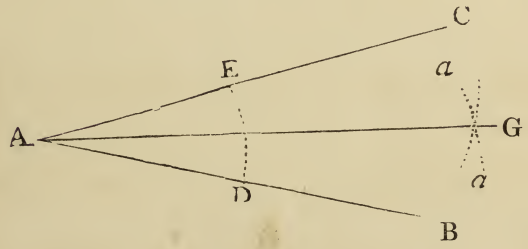
Take any two points, D and E, in the given line, with D as a centre, and radius D C describe an arc.

From E, with the radius E C, describe another.

Through the points of intersection C and F draw a line.—C G will be the perpendicular required.

PROBLEM IV.

To divide a given Angle B A C, *into two equal Parts.*



From the centre A, describe the arc D E.

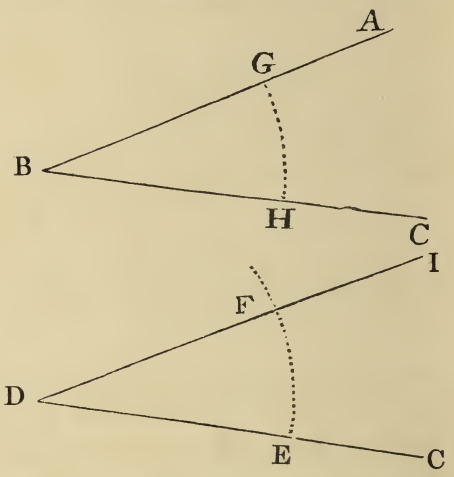
From the centre D, with any radius longer than half of D E, describe the arc *a a*.

From the centre E, with the same radius, describe another arc cutting the former in G.

Draw the line A G, which will divide the angle B A C into two equal parts.

PROBLEM V.

To make an Angle equal to a given Angle A B C.



Draw the line C D.

From D, as a centre, draw an arc E F.

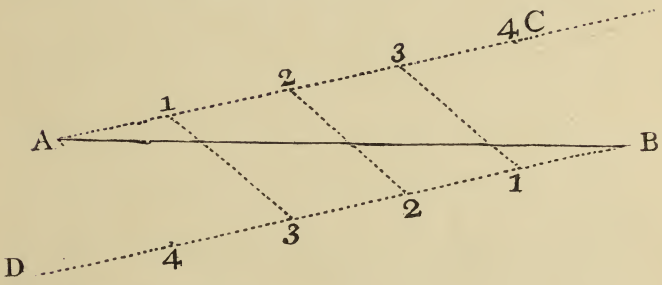
With the same radius, and from B, draw the arc G H.

Set the distance G H off from E to F.

Through F draw the line D I, and you will have the angle required.

PROBLEM VI.

To divide a given Line A B, *into any number of equal Parts.*



Draw the line A C.

Make an angle at B, equal to that at A.

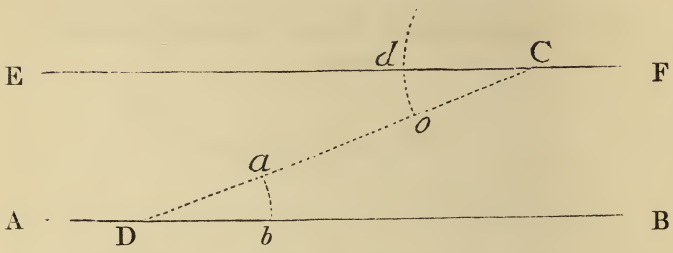
With any convenient distance, set off the number of parts required (suppose four) from A towards C.

Set off the same parts from B towards D.

Draw the lines (1...3), (2...2), &c., which will divide the line A B, as was required.

PROBLEM VII.

To draw a Line parallel to a given Line A B, *which shall pass through a given Point* C.



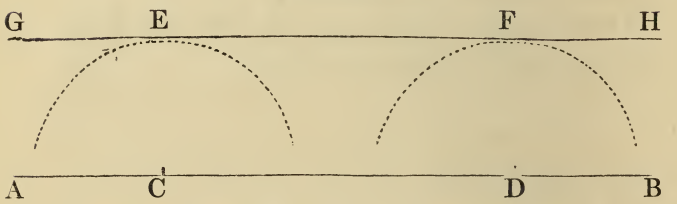
Take any point D, in the line A B, and draw the line D C.

Make the angle E C D equal to B D C.

The line E F will be parallel to A B.

PROBLEM VIII.

To draw a Line parallel to a given Line at a given Distance.



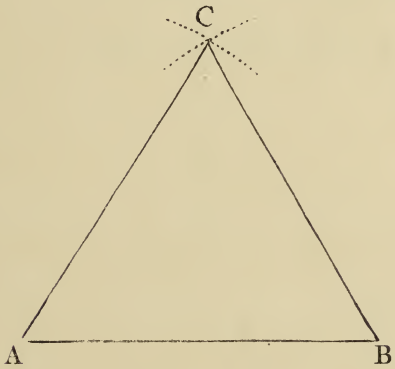
Take two points, C and D, in the given line.

Take the given distance for a radius, and, with C and D as centres, describe the arcs E and F.

Draw the line G H touching both arcs without cutting them, and it will be parallel to A B.

PROBLEM IX.

Upon a given right line A B, *to make an Equilateral Triangle.*



With the radius A B, and from A as a centre, describe an arc.

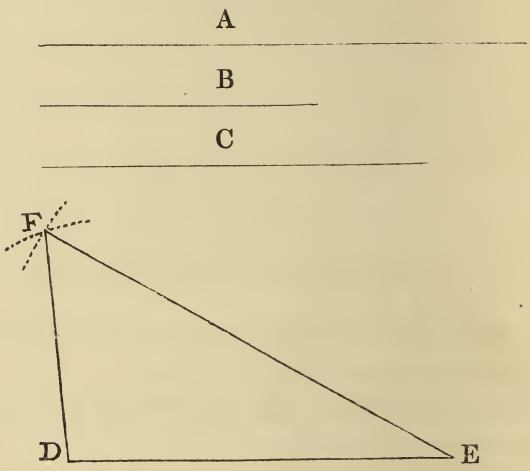
With the same radius A B, and from B as a centre, cross the first arc in C.

Draw the lines C A and C B, which will complete the triangle A B C.

PROBLEM X.

To make a Triangle whose three Sides shall be equal to three given straight lines, any two being greater than the third.

Let A B C *be the three given lines.*



Draw the line D E, equal to A.

With a radius equal to B, and from D as a centre, describe an arc.

With a radius equal to C, and from E as a centre, cross the first arc in F.

Draw the lines F D, F E, which will complete the triangle required.

PROBLEM XI.

To describe a Square upon a given Line A B.

Raise a perpendicular at one end of the given line.

Make the perpendicular B C equal to A B.

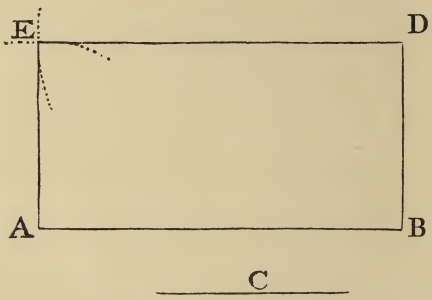
With the radius A B, and from C as a centre, describe an arc.

With the same radius, and from A as a centre, describe another arc, crossing the first in D.

Draw the lines D C, D A, which will complete the square required.

PROBLEM XII.

To describe a Rectangle, whose length and breadth shall be equal to two given Lines A B, *and* C.



Raise a perpendicular at one end of the longer line, as at B.

Make the perpendicular B D, equal to the shorter line C.

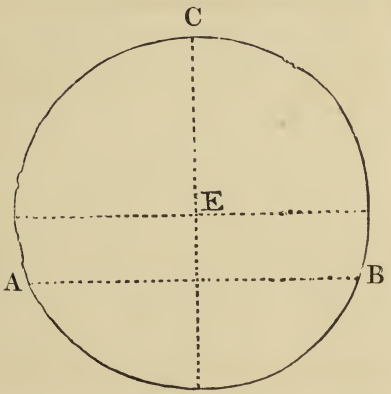
With the radius A B, and from D as a centre, describe an arc.

With the radius C, and from A as a centre, describe another arc, crossing the first in E.

Draw the lines E D and A E, which will complete the rectangle required.

PROBLEM XIII.

To find the Centre of a given Circle.

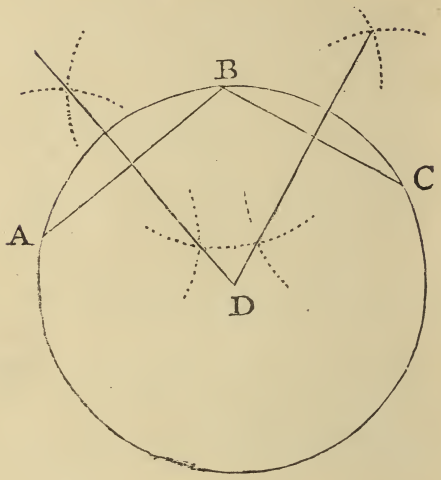


Draw any chord A B, and bisect it at right angles with the perpendicular C D.

Bisect C D in E, which will be the centre of the circle required.

PROBLEM XIV.

Through three given Points, A, B, *and* C, *to describe the Circumference of a Circle.*



From the middle point B, draw the lines B A and B C.

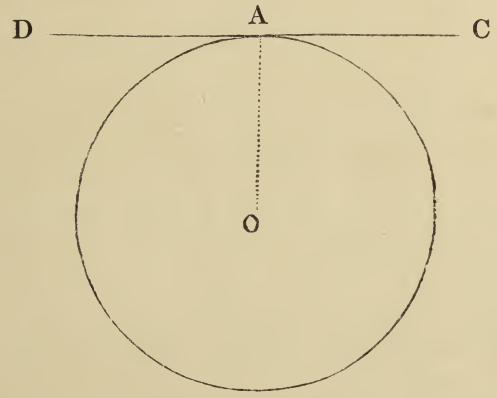
Bisect these two lines with two others drawn at right angles to them, and produce them till they meet in the point D.

With the radius D B, D C, or D A, and from D as a centre, describe a circle, which will pass through the points required.

PROBLEM XV.

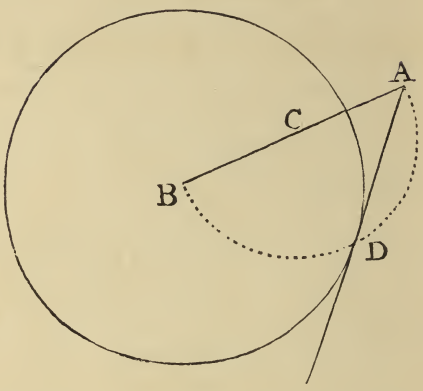
To draw a Tangent to a given Circle, that shall pass through a given Point A.

Case I.—*When the given point* A *is in the circumference of the circle.*



Draw the radius A O.
At the end of it A, erect a perpendicular, A C.
Produce the perpendicular to D.
C D will be the tangent required.

CASE II.—*When the given point* A *is without the circle.*



Draw the line A B from the given point to the centre of the circle B.

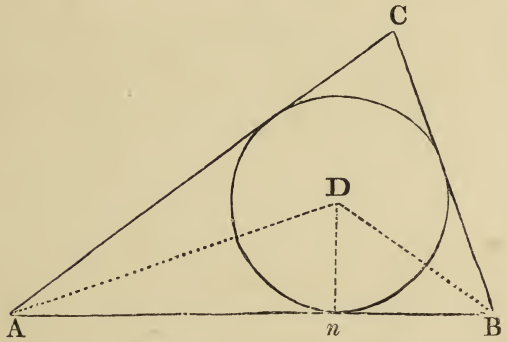
Bisect A B in C.

With the radius C B or C A, and from C as a centre, describe the semicircle A D B, cutting the given circle in D.

Draw A D, which will be the tangent required.

PROBLEM XVI.

To inscribe a Circle in a given Triangle A B C.

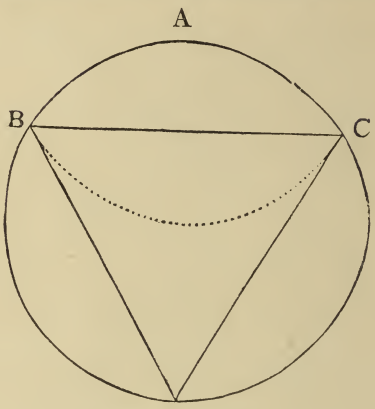


Bisect any two angles with two lines, these lines will intersect in the point D, which is the centre of the circle.

Then from the point of intersection D, let fall the perpendicular D *n*, and it will be the radius of the required circle.

PROBLEM XVII.

To inscribe an Equilateral Triangle in a given Circle.

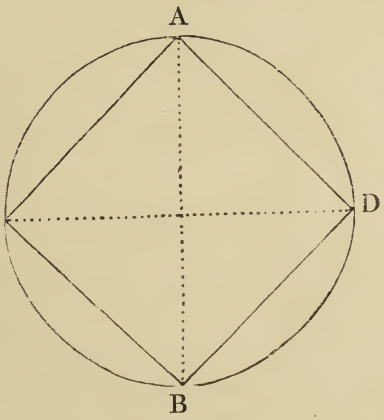


With the radius of the circle, and from any point A in the circumference as a centre, describe the arc B C.

Draw the line B C, which will be the side of the equilateral triangle required.

PROBLEM XVIII.

To describe a Square in a given Circle A B C D.

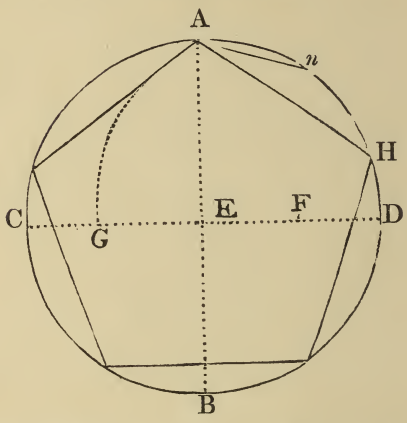


Draw two diameters A B and C D, crossing each other at right angles.

Draw the lines A D, D B, B C, and A C, which will form the square required.

PROBLEM XIX.

To inscribe a regular Pentagon, or a regular Decagon in a given Circle.



Draw two diameters, A B and C D, at right angles.

Bisect the radius E D in F.

With the radius F A, and from F as a centre, describe the arc A G.

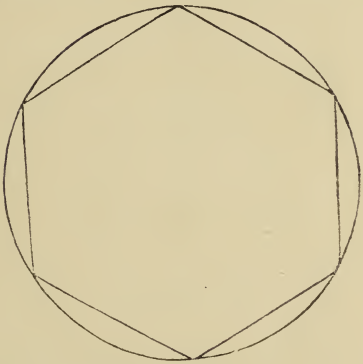
The distance A G is the side of the pentagon required.

For the Decagon.

Bisect the arc A H, subtending the side of the pentagon in *n*, and the line, A *n* being carried ten times round the circumference, will form the decagon required.

PROBLEM XX.

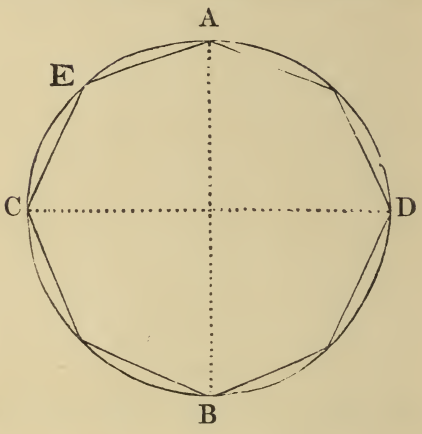
To inscribe a regular Hexagon in a given Circle.



Take the radius of the circle, and carry it six times round the circumferences.

PROBLEM XXI.

To inscribe a regular Octagon in a given Circle.



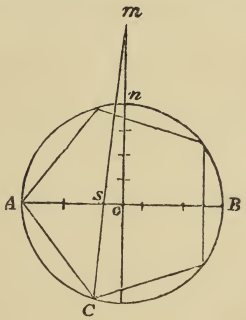
Draw two diameters A B and C D at right angles.

Bisect the arc A C in E.

Draw the line A E, which will be the side of the octagon.

PROBLEM XXII.

In a given Circle to inscribe any regular Polygon; or, to divide the Circumference of a given Circle into any number of equal Parts[1].



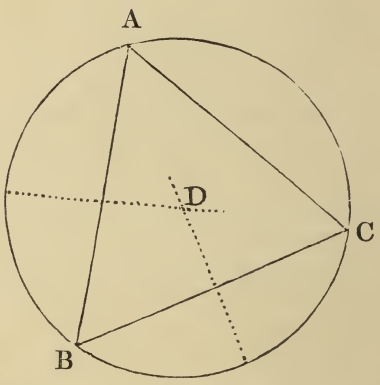
Divide the diameter A B into as many equal parts as the figure has sides; from the centre *o* draw the

[1] This problem can be performed accurately only in certain cases. M. Gauss, a German mathematician of great eminence, and professor of mathematics at Strasburgh, has shown that the geometrical division of the circle into equal parts may be effected in many cases, which were considered by mathematicians as impossible. Professor Gauss published, in 1801 (octavo), his celebrated work, in Latin, entitled *Disquisitiones Arithmeticæ*, and translated into French in 1807 (quarto), by M. Poullet Delisle, under the title of *Recherches Arithmétiques*. He has shown how a regular polygon of 17 sides may be described, or any other whose number of sides is denoted by 2^n+1, when this expression is a prime number. This work can be read with advantage only by the advanced student. For further information see Le Gendre's *Essaie sur la Théorie des Nombres*; Barlow on the *Theory of Numbers*; and Leybourn's *Mathematical Repository*.

perpendicular *o m*, divide the radius *o n* into four equal parts, and set off three of these parts from *n* to *m*; from *m*, through the second division, *s*, of the diameter A B, draw *m* C ; join A C, and it will be the side of the polygon required.

PROBLEM XXIII.

To describe a Circle about a given Triangle A B C.

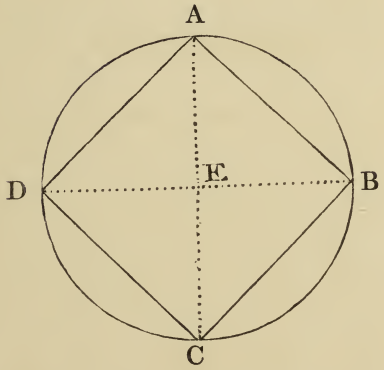


Bisect any two sides with two lines at right angles; these lines will intersect at the point D, which will be the centre of the circle.

With the radius D A, D B, or D C, describe a circle.

PROBLEM XXIV.

To describe a Circle about a given Square A B C D.



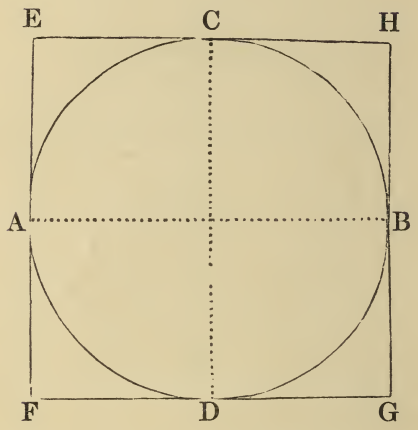
Draw the two diagonals A C and B D.

The intersection E is the centre, and the lines A E, B E, C E, and D E, are radii, with which draw the circle.

Note.—The centres of all regular polygons may be found by bisecting two adjoining sides, or two angles with two lines, which will intersect each other in the centre.

PROBLEM XXV.

To describe a Square about a given Circle.



Draw two diameters, A B and C D at right angles.

Through the point A draw a line parallel to C D.

Do the same at B.

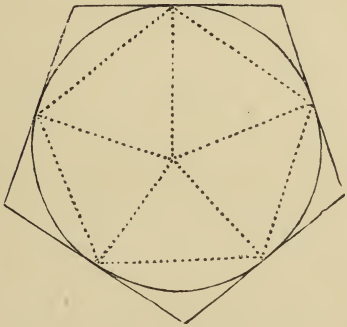
Through the point C draw a line parallel to A B.

Do the same at D.

These lines will intersect in the points E, F, G, H, and form a square.

PROBLEM XXVI.

To describe any regular Polygon about a Circle.

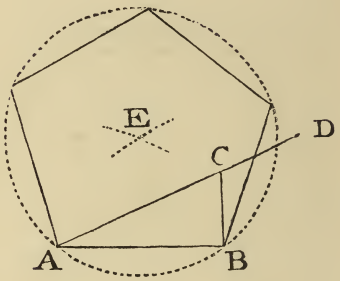


Inscribe a polygon with a given number of sides in the circle.

Draw tangents to the circle at the angular points, which will intersect each other, and form the polygon required.

PROBLEM XXVII.

To make a regular Pentagon on a given Line A B.



Draw B C perpendicular to A B, and equal to half of it.

Draw A C, and produce it till C D is equal to C B.

With the radius B D, and from A as a centre, describe an arc.

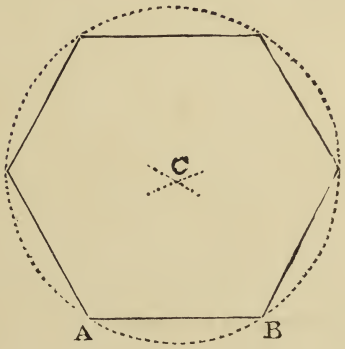
With the same radius, and from B as a centre, cross the first arc in E.

With the radius E A or E B, and from E as a centre, describe a circle.

Apply the given line five times round the circumference of the circle, and the pentagon will be formed.

PROBLEM XXVIII.

To make a regular Hexagon on a given Line A B.



With the radius A B, and from A as a centre, describe an arc.

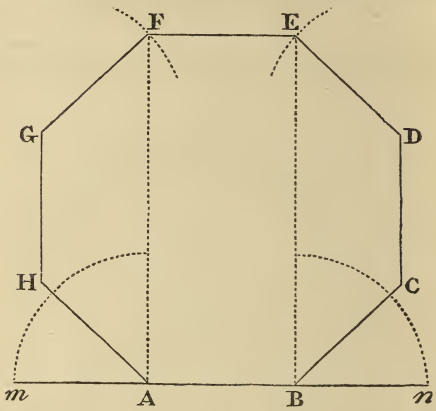
With the same radius, and from B as a centre, cross the first arc in C.

With the radius C A or C B, and from C as a centre, describe a circle.

Carry the line six times round the circumference of the circle, and the hexagon will be made.

PROBLEM XXIX.

On a given line A B *to describe a regular Octagon.*



On the ends of the given line A B erect the indefinite perpendiculars A F and B E.

Produce A B both ways to *m* and *n*.

Bisect the angles *m* A F and *n* B E with the lines A H and B C.

Make A H and B C each equal to A B.

Draw H G and C D parallel to A F or B E.

Make H G and C D each equal to A B.

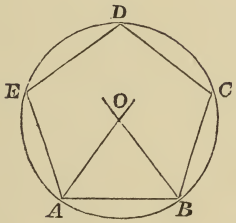
From the centre G, with the radius A B, cross the line A F in F.

With the centre D, and the same radius, cross the line B E in E.

Draw the lines G F, F E, E D, which will complete the octagon required.

PROBLEM XXX.

To make a regular Polygon on a given line A B.



Draw A O and B O, making the angles A and B each equal to half the angle of the polygon at the circumference. With the centre O and distance O A, describe a circle. Then apply the line A B continually round the circumference the proper number of times, and it is done.

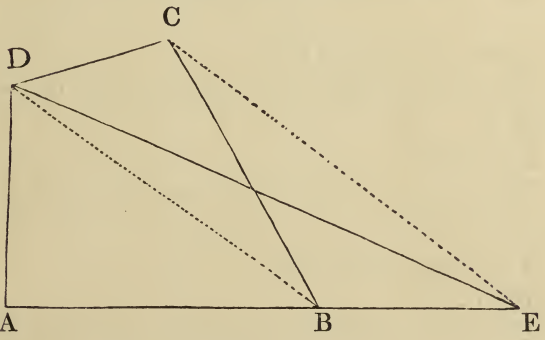
NOTE.—The angle of any polygon at the circumference, of which the angles O A B and O B A are each one-half, is found thus: Divide the whole 360 degrees by the number of sides of the given polygon, and the quotient will be the angle at the centre O; and this angle being subtracted from 180 degrees, the difference will be the angle of the polygon at the circumference, and is double of the angle O A B, or of O B A. According to this method the following table has been calculated, showing the angles at the centres and circumferences of regular polygons, from three to fifty sides inclusive:—

No. of Sides.	Names of the Polygons.	Angle O at the Centre.	Angles at the Circumference	Angle O A B, or O B A.
3	Trigon	120°.0′	60°.0′	30°.0′
4	Tetragon	90 .0	90 .0	45 .0
5	Pentagon	72 .0	108 .0	54 .0
6	Hexagon	60 .0	120 .0	60 .0
7	Heptagon	51 .25$\frac{5}{7}$	128 .34$\frac{2}{7}$	64 .17$\frac{1}{7}$
8	Octagon	45 .0	135 .0	67 .30
9	Nonagon	40 .0	140 .0	70 .0
10	Decagon	36 .0	144 .0	72 .0
11	Undecagon	32 .43$\frac{7}{11}$	147 .16$\frac{4}{11}$	73 .38$\frac{2}{11}$
12	Duodecagon	30 .0	150 .0	75 .0
13	Tridecagon..	27 .41$\frac{7}{13}$	152 .18$\frac{6}{13}$	76 .9$\frac{3}{13}$
14	Tetradecagon.....	25 .42$\frac{6}{7}$	154 .17$\frac{1}{7}$	77 .8$\frac{4}{7}$
15	Pentadecagon	24 .0	156 .0	78 .0
16	Hexadecagon.....	22 .30	157 .30	78 .45
17	Heptadecagon	21 .10$\frac{10}{17}$	158 .49$\frac{7}{17}$	79 .24$\frac{12}{17}$
18	Octadecagon	20 .0	160 .0	80 .0
19	Nonadecagon.....	18 .56$\frac{16}{19}$	161 .3$\frac{3}{19}$	80 .31$\frac{11}{19}$
20	Eicosagon	18 .0	162 .0	81 .0
21	Uneicosagon	17 .8$\frac{4}{7}$	162 .51$\frac{3}{7}$	81 .25$\frac{5}{7}$
22	Duoeicosagon	16 .21$\frac{9}{11}$	163 .38$\frac{2}{11}$	81 .49$\frac{1}{11}$
23	Triseicosagon.....	15 .39$\frac{3}{23}$	164 .20$\frac{20}{23}$	82 .10$\frac{10}{23}$
24	Tetreicosagon	15 .0	165 .0	82 .30
25	Penteicosagon	14 .24	165 .36	82 .48
26	Hexeicosagon	13 .50$\frac{10}{13}$	166 .9$\frac{3}{13}$	83 .4$\frac{8}{13}$
27	Hepteicosagon	13 .20	166 .40	83 .20
28	Octeicosagon.....	12 .51$\frac{3}{7}$	167 .8$\frac{4}{7}$	83 .34$\frac{2}{7}$
29	Noneicosagon	12 .24$\frac{24}{29}$	167 .35$\frac{5}{29}$	83 .47$\frac{17}{29}$
30	Triacontagon.....	12 .0	168 .0	84 .0
31	Untriacontagon.....	11 .36$\frac{24}{31}$	168 .23$\frac{7}{31}$	84 .11$\frac{19}{31}$
32	Duotriacontagon	11 .15	168 .45	84 .22$\frac{1}{2}$
33	Tritriacontagon.....	10 .54$\frac{6}{11}$	169 .5$\frac{5}{11}$	84 .32$\frac{8}{11}$
34	Tetratriacontagon....	10 .35$\frac{5}{17}$	169 .24$\frac{12}{17}$	84 .42$\frac{6}{17}$
35	Pentriacontagon	10 .17$\frac{1}{7}$	169 .42$\frac{6}{7}$	84 .51$\frac{3}{7}$
36	Hextriacontagon	10 .0	170 .0	85 .0
37	Heptriacontagon	9 .43$\frac{20}{37}$	170 .16$\frac{8}{37}$	85 .8$\frac{4}{37}$
38	Octriacontagon	9 .28$\frac{8}{19}$	170 .31$\frac{11}{19}$	85 .15$\frac{15}{19}$
39	Nonatriacontagon	9 .13$\frac{14}{13}$	170 .46$\frac{2}{13}$	85 .23$\frac{1}{13}$
40	Tesseracontagon	9 .0	171 .0	85 .30
41	Untesseracontagon ..	8 .46$\frac{34}{41}$	171 .13$\frac{7}{41}$	85 .36$\frac{24}{41}$
42	Duotesseracontagon ..	8 .34$\frac{2}{7}$	171 .25$\frac{5}{7}$	85 .42$\frac{6}{7}$
43	Tritesseracontagon ..	8 .22$\frac{14}{43}$	171 .37$\frac{29}{43}$	85 .48$\frac{36}{43}$
44	Tetratesseracontagon .	8 .10$\frac{10}{11}$	171 .49$\frac{1}{11}$	85 .54$\frac{6}{11}$
45	Pentesseracontagon ..	8 .0	172 .0	86 .0
46	Hextesseracontagon ..	7 .49$\frac{13}{23}$	172 .10$\frac{10}{23}$	86 .5$\frac{5}{23}$
47	Heptesseracontagon ..	7 .39$\frac{27}{47}$	172 .20$\frac{20}{47}$	86 .10$\frac{10}{47}$
48	Octesseracontagon ..	7 .30	172 .30	86 .15
49	Nonatesseracontagon .	7 .20$\frac{40}{49}$	172 .39$\frac{9}{49}$	86 .19$\frac{29}{49}$
50	Pentecontagon	7 .12	172 .48	86 .24

PROBLEM XXXI.

To reduce any Rectilinear Figure, which has more than three sides to another equal to it, but with one side less.—For example:—

To reduce the trapezium A B C D *to a triangle* A D E, *which shall be equal to it.*



Draw the diagonal D B.

Draw C E parallel to D B.

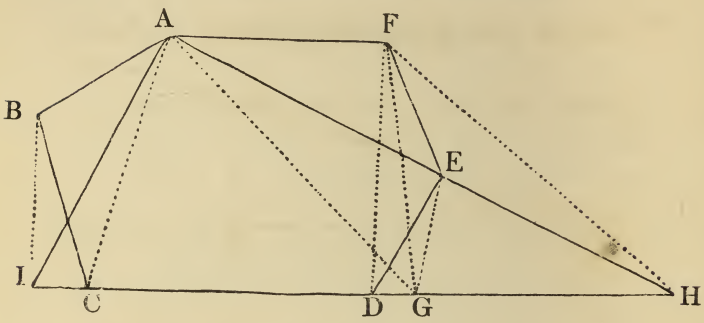
Produce A B till it meets C E in E.

Draw D E.

A D E will be the triangle required.

Hence any rectilinear figure may be reduced to a triangle, by reducing it successively to a figure with one side less, until it is brought to one with only three sides. For example:

To reduce the Polygon A B C D E F *to a Triangle* I A H, *which shall be equal to it.*



Draw the diagonal D F.

Draw E G parallel to D F till it meets C D produced.

Join F G.

The polygon A B C G F is equal to the given one A B C D E F.

To reduce A B C G F.

Draw A G.

Draw F H parallel to A G till it meets C G produced.

Join A H.

The polygon A B C H is equal to the one A B C G F.

To reduce A B C H.

Draw A C.

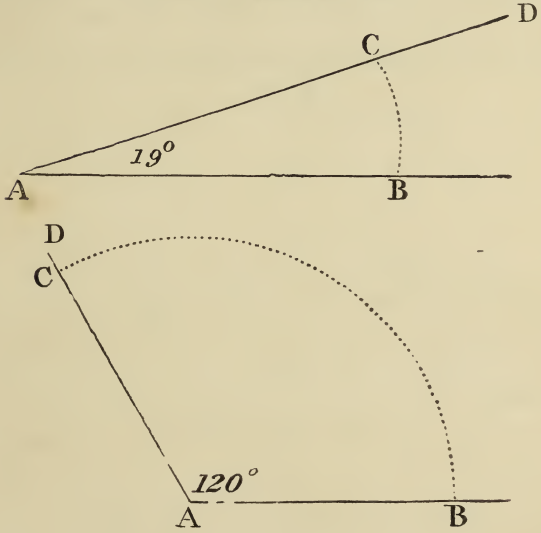
Draw B I parallel to A C till it meets H C produced.

Join A I.

The triangle A H I is equal to the given polygon A B C D E F.

PROBLEM XXXII.

To make an Angle of any proposed number of Degrees.



This and the following problem are most easily done by a protractor, for want of which draw the line A B.

Take the first 60 degrees from a scale of chords as a radius, and from A as a centre, describe an arc B C.

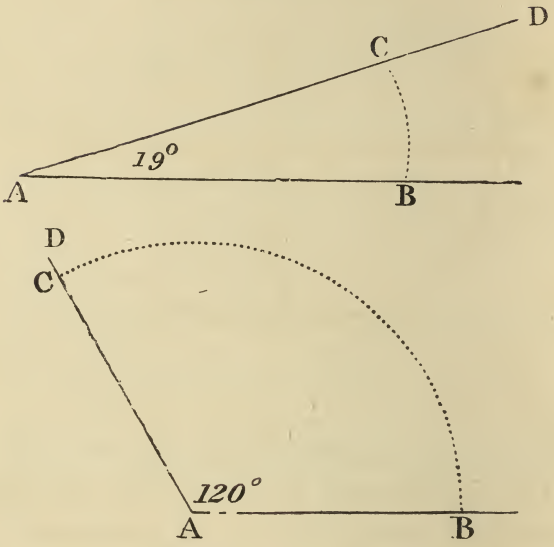
Take the proposed number of degrees from the scale of chords, and set off from B to C.

Draw the line A D, and the angle will be formed.

Note.—If the angle is to contain more than 90 degrees, it must be taken at twice.

PROBLEM XXXIII.

To find the Number of Degrees contained in an Angle B A C.



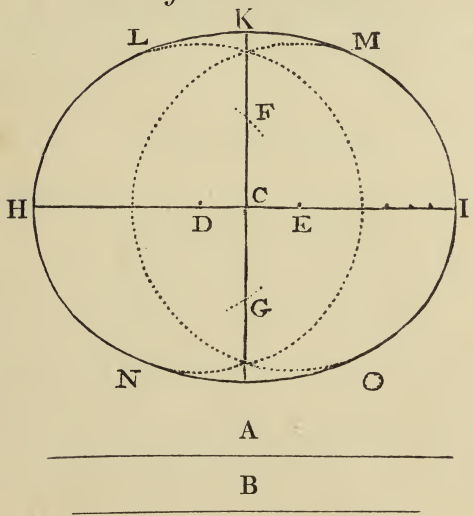
From A for a centre, and with a chord of 60 degrees for a radius, describe the arc B C.

Take the distance B C, and apply it to the scale of chords, which will show the number of degrees.

Note.—If the angle contains more than 90 degrees, it must be taken at twice.

PROBLEM XXXIV.

To draw an Oval, whose two Diameters shall be equal to two given Lines A *and* B.



Draw two diameters equal to the two given lines, crossing each other in the middle at right angles.

Take two thirds of the difference of the diameters, and set it off from the point of intersection C, on each side the longer line at D and E.

With the radius D E, and from D or E as a centre, cross the shorter diameter in F and G.

With the radius D H, and from D as a centre, describe a circle.

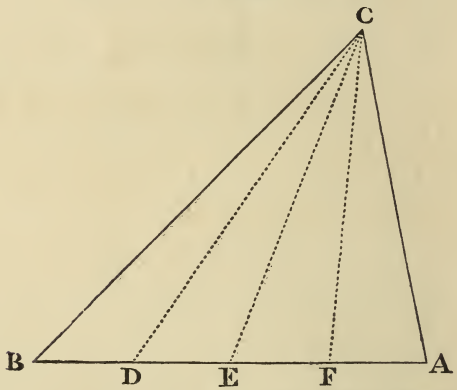
With the same radius, and from E as a centre, describe another circle.

With the radius G K, and from G as a centre, describe the arc L M.

With the same radius, and from F as a centre, describe the arc N O, which will complete the oval required.

PROBLEM XXXV.

To divide a Triangle A B C, *into any number of equal Parts (suppose four).*

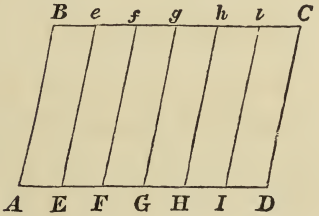


Divide the side A B opposite the vertex C into four equal parts, B D, D E, E F, F A.

From C, to the points of division D, E, F, draw the lines C D, C E, C F; then the triangle A B C is divided into four equal triangles C B D, C D E, C E F, C F A.

PROBLEM XXXVI.

To divide a Parallelogram A B C D *into a given number of equal Parts (suppose six) in such a way, that the Lines of Division may be parallel to* A B, *or* C D.

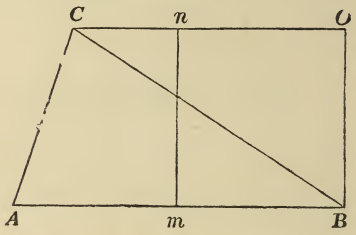


Divide A D into six equal parts A E, E F, F G, G H, H I, I D.

Draw E *e*, F *f*, G *g*, H *h*, I *i*, parallel to A B, or C D, then the division is done.

PROBLEM XXXVII.

To make a Rectangle, or Parallelogram, equal to a given Triangle A B C.



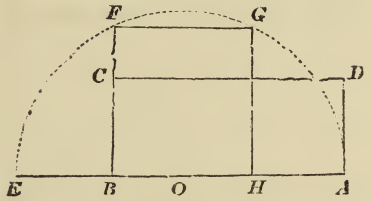
Bisect the base A B in *m*.

Through C draw C *n* O parallel to A B; and through *m* and B draw *m n* and B O parallel to each other, and either perpendicular to A B, or making any angle with it.

The rectangle or parallelogram *m n* O B is equal to the triangle A C B.

PROBLEM XXXVIII.

To make a Square equal to a given Rectangle A B C D.



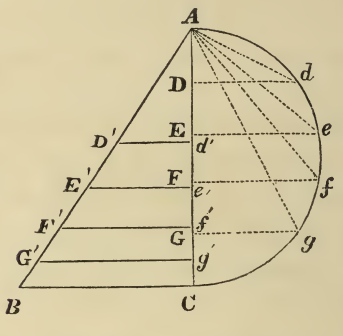
Produce the side A B till B E be equal to the other side B C.

Bisect A E in O, with which as a centre and radius A O, describe a semicircle, and produce B C to meet it in F.

On B F make the square B F G H, and it will be equal to the rectangle A B C D as required.

PROBLEM XXXIX.

To divide a given Triangle A B C, *into a given number of equal Parts (suppose five), and in such a way, that the Lines of Division may be parallel to* B C.



Upon A C, describe the semicircle A *d e f g* C, and divide A C, into five equal parts, A D, D E, E F, F G, G C.

Draw D *d*, E *e*, F *f*, G *g*, perpendicular to A C, meeting the semicircle in the points *d*, *e*, *f*, *g*.

Join A *d*, A *e*, A *f*, A *g*, then make A *d'*=A *d*, A *e'*=A *e*, A *f'*=A *f*, A *g'*=A *g*.

From the points *d'*, *e'*, *f'*, *g'*, draw the lines *d'* D', *e'* E', *f'* F', *g'* G', parallel to the side B C; then A D' *d'*, D' *d' e'* E', E' *e' f'* F', F' *f' g'* G', G' *g'* C B, are the five equal parts of the triangle A B C.

PART THE SECOND.

TO FIND THE AREA OF PLANE FIGURES.

THE area of any plane figure is the measure of its surface, or of the space contained within the boundaries of that surface, without any regard to thickness, and is estimated by the number of squares contained therein; the side of those squares being either an inch, a foot, a yard, a link, a chain, &c.

And hence the area is said to be so many square inches, square feet, square yards, square links, or square chains, &c.

The common measures of length are given in the first table below; the second table, of square measure, is taken from it, by squaring the several numbers; and the third by cubing them.

TABLE I.—OF LINEAL MEASURE.

This measure is used to measure distances, lengths, breadths, heights, depths, &c. of places or things.

Inches.	Gunter's Links.	Feet.	Yards.	Fathoms	Rods, Poles, or Perches.	Gunter's Chains.	Furlongs.	Mile.
$7\frac{23}{25}$	= 1							
12	$1\frac{17}{33}$	= 1						
36	$4\frac{6}{11}$	3	= 1					
72	$9\frac{1}{11}$	6	2	= 1				
198	25	$16\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{3}{4}$	= 1			
792	100	66	22	11	4	= 1		
7920	1000	660	220	110	40	10	= 1	
63360	8000	5280	1760	880	320	80	8	= 1

Besides the above, there are, the Barley-corn, which equals $\frac{1}{3}$ of an inch; the Palm=3 inches; the Hand=4 inches; the Span=9 inches; the Cubit=$1\frac{1}{2}$ foot; the common military Pace=

$2\frac{1}{2}$ feet; the geometrical Pace=5 feet; the League=3 miles; and 60 geographical or 69·114 English Miles (according to Colonel Mudge)= 1 degree.

A Rope=20 feet; a woodland Pole=18 feet; a forest or plantation Pole=21 feet; a Cheshire Pole=24 feet; a Sherwood Forest Pole=25 feet; and 1 Rood of fencing or ditching=7 yards.

TABLE II.—OF SQUARE MEASURE.

Square measure is used to measure all kinds of superficies; such as land, paving, flooring, plastering, roofing, slating, tiling, and every thing that has length and breadth.

Sq. Inches.	Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Poles Perches, or Rods.	Sq. chns.	Sq. Rood	Sq. Acres	Sq. Mile.
$62\frac{454}{625}$	= 1							
144	$2\frac{322}{1089}$	= 1						
1296	$20\frac{50}{121}$	9	= 1					
39204	625	$272\frac{1}{4}$	$30\frac{1}{4}$	=1				
627264	10000	4356	484	16	= 1			
1568160	25000	10890	1210	40	$2\frac{1}{2}$	= 1		
6272640	100000	43560	4840	160	10	4	= 1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640	= 1

A Hide of land=100 acres; a Yard of land=30 acres; a Square of flooring, partitioning, roofing, tiling, and slating, are each=100 square feet; a Rod of brick-work=272·25 square feet; in some places it is customary to allow 324 square feet to a Rod; sometimes a Rod=21 feet long, and 3 feet high=63 square feet.

TABLE III.—OF CUBIC, OR SOLID MEASURE.

This is used, in mensuration, to measure all kinds of solids, or such figures as consist of three dimensions, viz. length, breadth, and depth, or thickness.

Cubic Inches.	Cubic Feet.	Cubic Yards	Cubic Poles Rods, or Perches	Cub. Furlongs.	Cub. Mile.
1728	= 1				
46656	27	= 1			
7762392	$4492\frac{1}{8}$	$166\frac{3}{8}$	= 1		
496793088000	287496000	10648000	64000	= 1	
254358061056000	147197952000	5451776000	32768000	512	= 1

A Load of rough timber=40 cubic feet; a Load of squared timber=50 cubic feet; a Ton of shipping=42 cubic feet; a Floor of earth=324 cubic feet; a Cord of wood=8 feet long, 4 feet broad, and 4 feet deep=128 cubic feet; a Stack of wood=12 feet long, 3 feet broad, and 3 feet deep=108 cubic feet; a solid Yard of earth=1 load.

Land is measured by a chain, called *Gunter's Chain*, of four poles, or twenty-two yards in length, and consists of 100 equal links, each link being $\frac{22}{100}$ of a yard in length, or 7·92 inches. Ten square chains, or ten chains in length and one in breadth make an acre, or 4840 square yards, 160 square poles, or 100000 square links, each being the same in quantity.

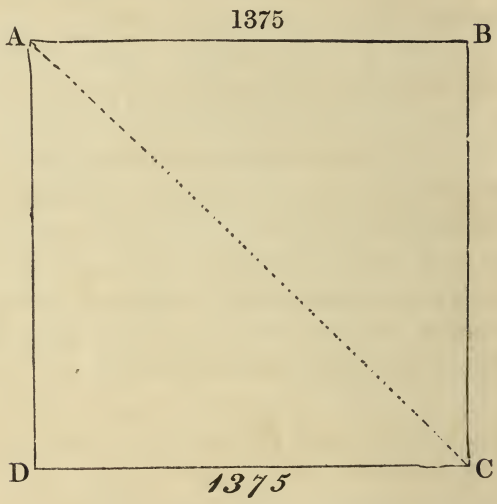
The length of lines measured with a chain are *generally* set down in links as whole numbers, every chain being 100 links in length. Therefore, after the dimensions are squared, or the superficies is found, it will be in square links; when this is the case, it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. These decimals must be then multiplied by 4 for roods, and the decimals of these again, after five figures are cut off, by 40, for perches.

PROBLEM I.

To find the Area of a Square.

RULE.—Multiply the side by itself and the product will be the area.

Example. What is the area of a square A B C D, whose side A B = 1375 links?



First. A B × B C = 1375 × 1375 = 1890625 square links the area. Then (1890625 ÷ 100000) × 4 × 40 = 18 acres, 3 roods, 25 perches, the content of the square A B C D = *Answer.*

Note 1. As the area is found by squaring the side, therefore, *conversely*, the side will be found by extracting the square root of the area. Likewise, the square root of the area of any plane figure, will give the side of a square equal in area to it.

Example. How long must the side of a square be, whose area A B C D=1890625 square links?

Here $\surd$ 1890625=1375 links the side of the square required.

Note 2. *To find the diagonal* A C, *or line joining the opposite corners.* Take the square root of twice the square of the side; or, which is the same thing, the square root of twice the area; or else multiply 1·4142136 (which is the square root of 2) by the side, the product will be the diagonal.

Example. Required the diagonal A C of the square A B C D, whose side A B=1375 links?

First. $\surd$ (2 A B^2)=$\surd$(2×1375^2)=$\surd$(2×1890625)=$\surd$3781250 =1944·5437 links, *nearly*=A C, the diagonal required.

Or thus,

1·4142136×1375=1944·5437 links=A C the diagonal, *as before.*

EXAMPLES FOR PRACTICE.

Example 1. Required the area of a square A B C D, whose side A B=25 chains. *Ans.* 62 *a.* 2 *r.*

Example 2. What is the area of a square A B C D, the side A B being 6½ chains? *Ans.* 4 *a.* 0 *r.* 36 *p.*

Example 3. The side of a square field A D= 1567 links, what is the area of the field A B C D?

Ans. 24 *a.* 2 *r.* 9 *p. nearly.*

Example 4. Required the area of a square field A B C D, whose side B C=3725 links.

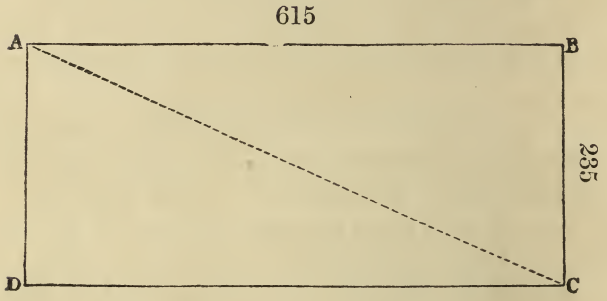
Ans. 138 *a.* 3 *r.* 1 *p.*

PROBLEM II.

To find the Area of a Rectangular Parallelogram.

RULE.—Multiply the length and breadth together, and the product will be the area.

Example. What is the area of a rectangular parallelogram, A B C D, its sides A B=615, and B C=235 links?



First. A B × B C = 615 × 235 = 144525 square links, the area.

Then (144525÷100000) × 4 × 40 = 1 acre, 1 rood, 31·24 perches, the content of the parallelogram A B C D required.

Note 1. If the area of a rectangular parallelogram be divided by one of its sides, the quotient will be the adjacent side.

Example. Let the area of the parallelogram A B C D be 144525 square links, and one of its sides A B=615 links, required the adjacent side A D.

Here A B C D÷A B=144525÷615=235 links, the length of the side A D, or B C required.

Note 2. The diagonal of a rectangular parallelogram is equal to the square root of the sum of the squares of any two adjacent sides.—*Euclid* 47 *of* 1.

Example. Required the diagonal A C of the rectangular parallelogram A B C D, whose adjacent sides are A B = 615, and B C = 235 links.

Here $\sqrt{(A\ B^2+B\ C^2)}=\sqrt{(615^2+235^2)}=\sqrt{(378225+55225)}=\sqrt{433450}=658{\cdot}369197$, &c. links=A C, the diagonal required.

EXAMPLES FOR PRACTICE.

Example 1. Required the area of a rectangular parallelogram, A B C D, whose two sides A B= 9025, and B C=60 links? *Ans.* 5 *a.* 1 *r.* 26·4 *p.*

Example 2. The length of a rectangular field A B=1225 links, and its breadth B C=613 links, required the area of the field A B C D.

Ans. 7 *a.* 2 *r.* 1·48 *p.*

Example 3. What is the area of a rectangular piece of ground A B C D, the length A B=1375 links, and breadth B C=950 links?

Ans. 13 *a.* 0 *r.* 10 *p.*

Example 4. Required the area of a rectangular court-yard A B C D, whose length A B=46$\frac{1}{3}$ feet, and breadth B C=15$\frac{1}{2}$ feet. *Ans.* 718$\frac{1}{6}$ ft.

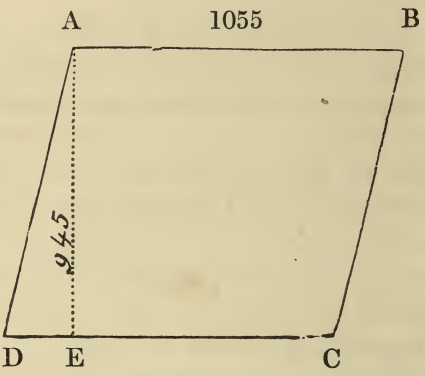
Example 5. Required the number of square yards in a rectangle A B C D, the length A =37 feet, and breadth A D=5$\frac{1}{4}$ feet. *Ans.* 1 $_{1}\frac{7}{2}$ yds.

PROBLEM III.

To find the Area of a Parallelogram, whether it be a Rhombus or Rhomboides.

RULE.—Multiply the length by the perpendicular height, and the product will be the area.

Example. What is the area of a rhombus, whose length A B=1055 links, and perpendicular A E= 945?



First. A B × A E = 1055 × 945 = 996975 square links, the area.

Then (996975÷100000) × 4 × 40 = 9 acres, 3 roods, 35·16 poles, the content of the rhombus A B C D required.

Note 1. The perpendicular breadth of any parallelogram will be found by dividing the area by the length; and the length will be found by dividing the area by the breadth. To find the diagonals, the angles must be known, and the operation will require Plane Trigonometry.

Example. Required the breadth A E of the rhombus A B C D, whose area is 996975 square links, and its length A B=1055 links.

First, A B C D÷A B=996975÷1055=945 links=AE, the breadth

required; and *conversely*, to find the length A B C D ÷ A E= 996975÷945=1055 links=A B, the length.

Note 2. The diagonals of a rhombus (the same as the square) being given, the area may be found by multiplying the diagonals together, and half the product will be the area.

Example. The diagonals of a rhombus being 30 and 20 chains, required the area.

Here $\frac{1}{2}$ (30 × 20) = 300 chains, which ÷ by 10=30 acres= *Answer.*

EXAMPLES FOR PRACTICE.

Example 1. Find the area of a rhombus A B C D, whose length D C=45$\frac{1}{2}$ chains, and perpendicular A E=40 chains. *Ans.* 182 *acres.*

Example 2. Required the area of a rhomboid A B C D, whose base D C=6, and the perpendicular height A E=4 chains. *Ans.* 2 *a.* 1 *r.* 24 *p.*

Example 3. Required the area of a rhombus A B C D, whose length A B=6·2 chains, and perpendicular height A E=5·45 chains.

Ans. 3 *a.* 1 *r.* 20·64 *p.*

Example 4. What is the area of a rhombus A B C D, whose length A B=725 links, and perpendicular height A E=635 links?

Ans. 4 *a.* 2 *r.* 16·6 *p.*

Example 5. A rhomboid A B C D, whose dimensions are as follows: A B=4784 links, and perpendicular height A E=1908 links, required the area.

Ans. 91 *a.* 1 *r.* 4·5952 *p.*

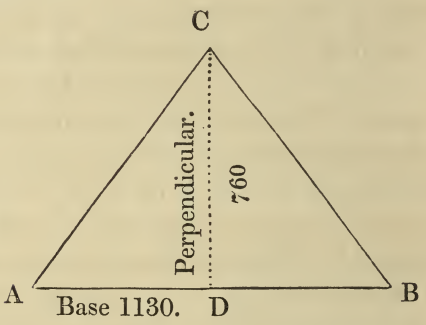
Example 6. How many square yards are there in a rhomboid A B C D, whose length A B=57$\frac{1}{2}$ feet and breadth A E=5$\frac{1}{4}$ feet? *Ans.* 33$\frac{1}{2}\frac{3}{4}$ yards.

PROBLEM IV.

To find the Area of a Triangle, by having the base and perpendicular given.

RULE.—Multiply any one of its sides by a perpendicular, let fall upon it from its opposite angle, and half the product will be the area; or multiply half one by the other, and the product will be the area.

Example. What is the area of a triangle A B C, whose base A B=1130, and perpendicular C D= 760 links?



Here $\frac{1}{2}$ (A B × C D) = $\frac{1}{2}$ (1130 × 760) = 858800 ÷2 = 429400 square links, the area.

Then (429400 ÷ 100000) × 4 × 40 = 4 acres, 1 rood, 7·04 poles, the content of the triangle A B C required.

Note 1. If the area of a triangle be divided by half the perpendicular height, the quotient will be the base. If the area be divided by half the base, the quotient will be the perpendicular height.

Example 1. If the area of the triangle A B C=429400 square links,

and its perpendicular height C D=760 links, required the length of the base A B.

Here A B C÷½ C D = 429400÷(760÷2) = 429400÷380 = 1130 links=A B, the base required.

Example 2. The area of the triangle A B C=429400, as in last example, and the base A B=1130, required the height of the perpendicular D C.

Here A B C D÷½ A B=429400÷(1130÷2)=429400÷565=760 links=D C, the perpendicular required.

Note 2. The side of a square inscribed in a triangle equals the product of the base and altitude, divided by their sum.

Example. Let the base of the triangle=12, and its altitude=4, required the side of the inscribed square.

The side of the inscribed square=(12×4)÷(12+4)=48÷16=3.

EXAMPLES FOR PRACTICE.

Example 1. What is the area of a triangle A B C, whose base A B=145, and perpendicular C D=84, yards? *Ans.* 1 *a.* 1 *r.* $1\frac{30}{121}$ *p.*

Example 2. What is the area of a triangle A B C, whose base A B=47 chains, and perpendicular D C $=25\frac{1}{2}$? *Ans.* 59 *a.* 3 *r.* 28 *p.*

Example 3. Required the area of a triangle A B C, whose base A B=575 links, and perpendicular C D =240. *Ans.* 2 *r.* 30·4 *p.*

Example 4. The base of a triangular field A B= 1236 links, and perpendicular C D=731 links, required the content of the triangle A B C.

Ans. 4 *a.* 2 *r.* 2·8128 *p.*

Example 5. How many square yards are contained in a triangle whose base A B=49 feet, and its perpendicular height C D=$25\frac{1}{4}$ feet?

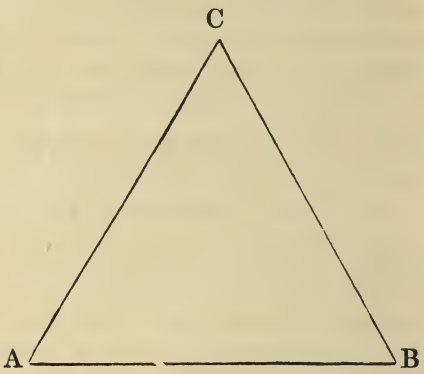
Ans. $68\frac{5}{7}\frac{3}{2}$, or 68·7361′ *sq. yds.*

PROBLEM V.

To find the Area of a Triangle by having the three sides given.

From half the sum of the three sides subtract each side separately from the half sum.

Then multiply the half sum and the three remainders continually together, and the square root of the product will be the area.



Example. Required the area of the triangle A B C, whose three sides A B = 13, B C = 14, and A C = 15 chains respectively.

First $\frac{1}{2}$ (A B + B C + A C) = $\frac{1}{2}$ (13 + 14 + 15) = 42 ÷ 2 = 21 chains, the half sum of the sides.

$$\left.\begin{array}{l} 21-13=8 \\ 21-14=7 \\ 21-15=6 \end{array}\right\} \text{the three remainders.}$$

Then $\sqrt{(21\times 8\times 7\times 6)} = \sqrt{7056} = 84$ square chains, the area; hence $(84 \div 10) \times 4 \times 40 = 8$ acres, 1 rood, 24 perches, the content of the triangle A B C.

Note 1.—*If the triangle be equilateral*, the area will be found* by multiplying one-fourth the square of a side by 1·732; or, which is the same thing, multiply the square of a side by ·433; CONVERSELY. *If the area of an equilateral triangle be given to find its side.* Divide the given area by ·433, the square root of the quotient is the side required.

Example 1. Let the side of an equilateral triangle be 12, required the area.

Here $12^2 \times \cdot 433 = 144 \times \cdot 433 = 62 \cdot 352$, the area required.

Example 2. If the area of an equilateral triangle be 62·352, required the side.

Here $\sqrt{62 \cdot 352} \div \cdot 433 = \sqrt{144} = 12$, the side required.

Note 2. As the perpendicular height will be found by dividing the area of a triangle by half its base (see *Note* 1, *Prob.* IV.), we shall, in the case of the equilateral triangle, have the perpendicular by multiplying the base by ·866.

Example. The base of an equilateral triangle is 12, required the perpendicular.

Here $12 \times \cdot 866 = 10 \cdot 392$, the perpendicular required.

Note 3. *When the two equal sides, and the third side of any isosceles triangle are given to find the area.* Multiply the sum of one of the equal sides, and half the third side by their difference, and the square root of the product by half the third side, the result will be the area.

Example. Let the two equal sides of an isosceles triangle be each equal 6 feet, and the third side equal 4 feet, required the area. Here $2\sqrt{[(6+2)\times(6-2)]} = 2\sqrt{(8\times 4)} = 2\sqrt{32} = 5 \cdot 6568 \times 2 = 11 \cdot 3137$ square feet, the area required.

Note 4. First. *Given the three sides of a triangle to find the segments of the base made by a perpendicular from the vertex of the opposite angle.* —Add the square of the base to the difference of the squares of the two sides, the sum divided by twice the base will give the greater segment, and this subtracted from the whole base will give the less segment.

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* The side of an equilateral triangle divided by 1·732 (which is the square root of 3), gives the radius of its circumscribing circle.—*See Keith's Geometry, Edited by S. Maynard, Book* VII. *Prob.* 187.

Or thus,

Divide the product of the sum and difference of the two sides by twice the base, this quotient added to half the length of the base will give the greater segment; and subtract the same quotient from the whole base, will give the less segment.

Second. *To find the perpendicular.*—By the foregoing process (*see diagram to Prob.* IV.) the triangle A B C is divided into two right-angled triangles, A D C, B D C, in each of which two sides, A C, A D, or B C, B D, being given, the perpendicular will be obtained (*by Rule* 2, *Prob.* VI.).

Example 1. (*See the Diagram to Prob.* IV.) Suppose the three sides of a triangle to be A C=31, B C=20, and A B=43, to find the greater segment of the base made by the perpendicular C D.

Here $\frac{A\,B^2+(A\,C^2-B\,C^2)}{2\,A\,B}=\frac{43^2+(31^2-20^2)}{43\times 2}=\frac{1849+(961-400)}{86}$

$=\frac{1849+561}{86}=2410\div 86=28\frac{1}{43}$, the greater segment A D required; hence, A B − A D = $43-28\frac{1}{43}=14\frac{42}{43}$, the less segment B D.

Or thus,

$\frac{(A\,C+B\,C)\times(A\,C-B\,C)}{2\,A\,B}+\frac{A\,B}{2}=\frac{(31+20)\times(31-20)}{43\times 2}+\frac{43}{2}=$

$\frac{561}{86}+\frac{1849}{86}=\frac{1205}{43}=28\frac{1}{43}$, the greater segment A D, *as before.*

Example 2. To find the perpendicular C D.

By Rule II. *Prob.* VI. $\sqrt{(B\,C^2-B\,D^2)}=\sqrt{[20^2-(14\frac{42}{43})^2]}=\sqrt{(400-224{\cdot}3)}=\sqrt{175{\cdot}7}=13{\cdot}25$, the perpendicular C D required.

EXAMPLES FOR PRACTICE.

Example 1. How many acres are contained in a triangle whose three sides are A B=2564, B C=2345, and A C=2139 links? *Ans.* 23 *a.* 2 *r.* 0·584 *p.*

Example 2. The sides of a triangular field are A B =4900, B C=5025, and A C = 2569 links; required the area. *Ans.* 61 *a.* 1 *r.* 39·68 *p.*

Example 3. How many acres are contained in a triangular fish-pond, whose three sides are A B=293, B C=239, and A C=185 yards; and what did the ground which it occupies cost at £185 per acre?

Ans. { 22064·7454 square yards, the area of the fish-pond A B C; and
£843. 7*s.* 8·1264*d.* the price of the ground.

Example 4. A field of a triangular form, whose sides are A B=380, B C=420, and A C=765 yards, lets for 55*s.* per acre; what is the annual rent?

Ans. { 9·235337 acres, the area of the field A B C; and
£25. 7*s.* 11¼*d.* ·28968 the annual rent.

PROBLEM VI.

Any two Sides of a Right-angled Triangle being given, to find the third Side.

RULE 1.—*When the base and perpendicular are given, to find the hypothenuse*.* To the square of the base add the square of the perpendicular; the square root of the sum will give the hypothenuse, or longer side.

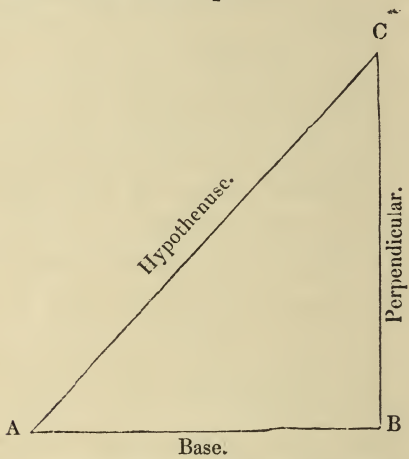
RULE 2.—*When the hypothenuse and one side are given, to find the other side*.* Multiply the sum of

* The truth of these Rules is evident from *Euclid* 47 *of* 1.

the hypothenuse, and one side by their difference; the square root of the product will give the other side.

Or thus,

From the square of the hypothenuse subtract the square of the given side; the square root of the remainder will be the side required.



Example 1. The perpendicular of a right-angled triangle B C=24 chains, the base A B=18 chains; what is the length of the hypothenuse A C?

By Rule 1. $\sqrt{(B\,C^2 + A\,B^2)} = \sqrt{(24^2 + 18^2)} = \sqrt{(576+324)} = \sqrt{900} = 30$ chains, the length of the hypothenuse A C required.

Example 2. The hypothenuse of a right-angled triangle A C=30 chains, and the perpendicular B C=24 chains; what is the length of the base A B?

By Rule 2. $\sqrt{[(A\,C+B\,C)\times(A\,C-B\,C)]} = \sqrt{[(30+24)\times(30-24)]} = \sqrt{(54\times6)} = \sqrt{324} = 18$ chains, the length of the base A B required.

Or thus,

$\sqrt{(A\,C^2-B\,C^2)}=\sqrt{(30^2-24^2)}=\sqrt{(900-576)}=\sqrt{324}=18$ chains, the length of the base A B, *as before.*

Note 1. *To find the perpendicular and base of a right-angled triangle, by having its area and hypothenuse given.*

RULE. *First.* Take half the square root of the sum of the square of the hypothenuse, and four times the area of the triangle.

Second. Take half the square root of the difference of the hypothenuse, and four times the area of the triangle.

Third. Then the sum and difference of the two preceding results will give the sides.

Example. The area of the right-angled triangle A B C=216 square yards, and the hypothenuse A C=30 yards, required the length of the perpendicular B C, and the base A B.

Here $\frac{1}{2}\sqrt{(A\,C^2+4\,A\,B\,C)}\pm\frac{1}{2}\sqrt{(A\,C^2-4\,A\,B\,C)}=\frac{1}{2}\sqrt{[30^2+(4\times216)]}\pm\frac{1}{2}\sqrt{[30^2-(4\times216)]}=\frac{1}{2}\sqrt{(900+864)}\pm\frac{1}{2}\sqrt{(900-864)}=\frac{1}{2}\sqrt{1764}\pm\frac{1}{2}\sqrt{36}=\frac{1}{2}(42\pm6)=24$ yards and 18 yards=B C and A B, the length of the two sides required.

Note 2. The radius of the circle inscribed in a right-angled triangle equals half the difference between the hypothenuse, and the sum of the base and perpendicular.

Example. The base of a right-angled triangle A B=18, the perpendicular B C=24, and the hypothenuse A C=30; required the radius of the circle inscribed in it.

Here $\frac{1}{2}[(B\,C+A\,B)\sim A\,C]=\frac{1}{2}[(24+18)\sim30]=\frac{1}{2}(42\sim30)=6$, the radius required.

Note 3. If half the sum of the three sides of any plane triangle be multiplied by the radius of the inscribed circle, the product will be the area; and, consequently, if the area be divided by half the sum of the sides, the quotient will be the radius of the inscribed circle.

Example 1. Let the three sides of any plane triangle be 41, 29, and 56 yards, and the radius of the inscribed circle 9·1165 yards, required the area of the triangle.

Here $\frac{1}{2}(41+29+56)\times9{\cdot}1165=63\times9{\cdot}1165=574{\cdot}3395$ square yards, the area of the triangle required.

Example 2. If the area of any plane triangle be 574·3395 square yards, and half the sum of the sides = 63 yards, the radius of the inscribed circle is required.

Here 574·3395÷63=9·1165 yards, the inscribed radius required.

Note 4. If the product of the three sides of any plane triangle be divided by four times the Radius of the circumscribing circle, the quotient will be the area; and, consequently, if the product of the sides be divided by four times the area, the quotient will be the Radius of the circumscribing circle.

Example 1. If the three sides of any plane triangle are 41, 29, and 56 yards respectively, and the Radius of the circumscribing circle be 28·98286 yards, required the area of the triangle.

Here $(41 \times 29 \times 56) \div (4 \times 28\,98286) = 66584 \div 115{\cdot}93144 =$ 574·3395 square yards, the area required.

Example 2. Let the three sides of any plane triangle be 41, 29, and 56 yards, and the area 574·3395 square yards, required the Radius of the circumscribing circle.

Here $(41 \times 29 \times 56) \div (4 \times 574{\cdot}3395) = 66584 \div 2297{\cdot}358 =$ 28·98286 yards, the circumscribed Radius required.

EXAMPLES FOR PRACTICE.

Example 1. The hypothenuse of a right-angled triangle A C=24, and the base A B=13, what is the length of the perpendicular B C? *Ans.* 18.

Example 2. Two ships set sail from the same port (suppose from B); one of them sails due West (B A), 50 miles; the other due North (B C) 84 miles; how many miles are they from each other?

Ans. 97·754795 miles=A C.

Example 3. A line 27 yards long will reach from he top of a fort on the opposite bank of a river,

to the water edge on this side of the river; what is the height of the fort, the river being 24 yards across? *Ans.* 12·369317 yards.

PROBLEM VII.

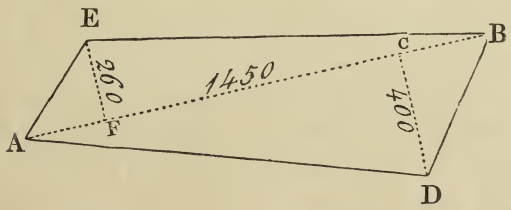
To find the Area of a Trapezium.

RULE.—Divide the trapezium into two triangles by a diagonal drawn from two of its opposite angles, and the sum of the areas of these triangles will be the answer.

Or,

If a perpendicular to each triangle will fall on the diagonal, multiply the sum of these perpendiculars by the diagonal, and half the product will be the area.

Example. Required the area of the following trapezium A E B D, the dimensions A B=1450, E F=260, C D=400 links.



Here $\frac{1}{2}$ [(E F + C D) × A B] = $\frac{1}{2}$ [(260 + 400) × 1450] = 957000 ÷ 2 = 478500 square links, the area.

Hence (478500 ÷ 100000) × 4 × 40 = 4 acres, 3 roods, 5·6 perches, the dimensions of the trapezium A E B D required.

Note. If the trapezium can be inscribed in a circle, the sum of its opposite angles will be equal to two right angles, (EUCLID 22 of III.) and its area may be found by the following rule, viz.: Add the four sides together, and take half the sum, from which half sum subtract each side separately. The square root of the product of the four remainders will give the area of the trapezium.

Example. What is the area of a trapezium inscribed in a circle, whose four sides are 24, 26, 28, and 30 yards?

$\frac{1}{2}(24+26+28+30)=54$ yards, the half sum of the sides.

Then $\sqrt{[(54-24)\times(54-26)\times(54-28)\times(54-30)]} = \sqrt{(30\times 28\times 26\times 24)} = \sqrt{524160} = 24\sqrt{910} = 24\times 30{\cdot}1662062 = 723{\cdot}9889488$ square yards, the area of the inscribed trapezium required.

EXAMPLES FOR PRACTICE.

Example 1. Required the area of a trapezium, the diagonal A B=750 links, and the perpendiculars E F=350 and C D=475. *Ans.* 3 *a.* 0 *r.* 15 *p.*

Example 2. A field, in form of a trapezium, has a diagonal A B=645 links, and the perpendiculars C D=350, and E F=200, required its area.

Ans. 1 *a.* 3 *r.* 3·8 *p.*

Example 3. Find the content of a trapezium, whose diagonal A B=1040 links, and the two perpendiculars C D=330, and E F=240 links.

Ans. 2 *a.* 3 *r.* 34·24 *p.*

Example 4. Suppose in the trapezium A E B D, on account of obstacles, I could only measure as follows, viz. the diagonal A B=378 yards, the side B D=220 yards, the side A E=265 yards; but it

is known that the perpendicular C D will fall 100 yards from B; and the perpendicular E F will fall 70 yards from A; required the area in acres.

Answer. The perpendicular C D=195·9591 yards; E F=255·5875 yards; and the area of the trapezium A E B D=85342·3074 square yards, or 17 *a.* 2 *r.* 21·232 *p.*

Example 5. It is required to draw the plan and find the area of a four-sided field A B C D A *(by Rule to Prob.* V.*)*, whose dimensions are as follow: south side D C=2740 links, east side B C=3575 links, north side A B=3755 links, west side A D=4105 links, and the diagonal from south-west to north-east, D B=4835 links.

Square Links.

The area of the north-west triangle, A B D=7470929·78
The area of the south-east triangle, B D C=4836484·00

The sum = 12307413·78

square links=123 *a.* 0*r.* 11·862048 *p.*=*Ans.* - - -

Example 6. The four sides of a trapezium inscribed in a circle are 800, 610, 508, and 420 feet; required the area of the trapezium. *Ans.* 319566 sq. feet.

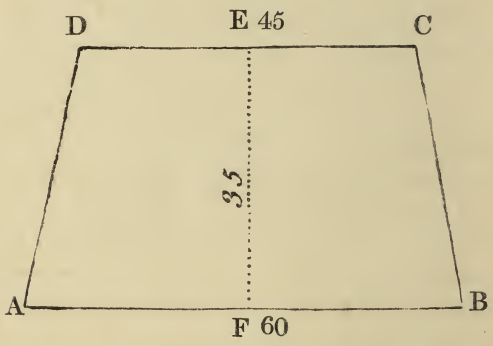
Example 7. The four sides of a trapezium inscribed in a circle are 427, 517, 620, and 810 links; what is the area of the trapezium? *Ans.* 11366630 sq. links.

PROBLEM VIII.

To find the Area of a Trapezoid.

RULE.—Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.

Example. How many links are there in a trapezoid, the parallel sides being D C = 45, and A B = 60, and the distance E F = 35 links?



Here $\frac{1}{2}$ (D C + A B) × E F = $\frac{1}{2}$ (45 + 60) × 35 = $\frac{1}{2}$ (105 × 35) = 3675 ÷ 2 = 1837·5 square links = *Answer.*

Note. If the area of a trapezoid and the sum of the parallel sides be given, the perpendicular distance between them will be found by dividing the area by half the given sum of the parallel sides.

Example. The parallel sides of a trapezoid being A B = 60 links,

and D C = 45 links, also the area A B C D = 1837·5 square links; required the breadth E F.

Here A B C D $\div \frac{1}{2}$ (A B + D C) = 1837·5 $\div \frac{1}{2}$ (60 + 45) = 1837·5 $\div$ 52·5 = 35 links, the breadth E F required.

EXAMPLES FOR PRACTICE.

Example 1. Required the area of a trapezoid, the two sides being A B=10, and C D=4 chains, and the distance E F=$1\frac{1}{2}$. *Ans.* $10\frac{1}{2}$ chains.

Example 2. What is the area of a trapezoid, the two sides being A B=75, and C D=20 links, and the distance E F=240? *Ans.* 11400 links.

Example 3. A field in the form of a trapezoid, whose parallel sides are A B=6340, and C D=4380 yards, and the perpendicular distance between them E F=121 yards, lets for 207*l.* 14*s.* per annum, what is that per acre?

Ans. { 134 acres, the area of the field, and 1*l.* 11*s.* the yearly rent per acre.

Example 4. A field in the form of a trapezoid, whose parallel sides are D C = 220, and A B = 1600 links, the perpendicular distance between them E F = 2544 links, lets for 2*l.* 10*s.* per acre; it is required to find the area of the field, and also its annual rent?

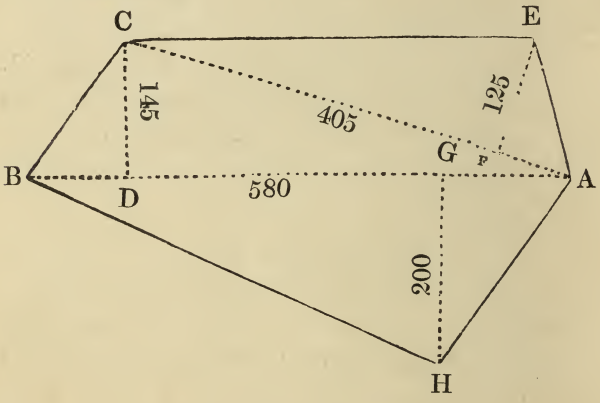
Ans. { The area = 23 *a.* 0 *r.* 24·064*p.*; and the annual rent = 57*l.* 17*s.* 6·24*d.*

PROBLEM IX.

To find the Area of an irregular Polygon.

RULE.—Draw diagonals, dividing the figure into trapeziums, or trapezoids and triangles, then find the areas of each of these separately by the preceding problems, add these areas together, and the sum will be the area of the polygon.

Example 1. Required the content of the following irregular polygon B C E A H, the dimensions A B=580, G H=200, C D=145, A C=405, E F=125, being links.



By Rules to Problem IV. *and* VII.

Square Links.

(C D + G H) × B A = (145 + 200) × 580 - =200100

C A × E F = 405 × 125 - - - - - - - - = 50625

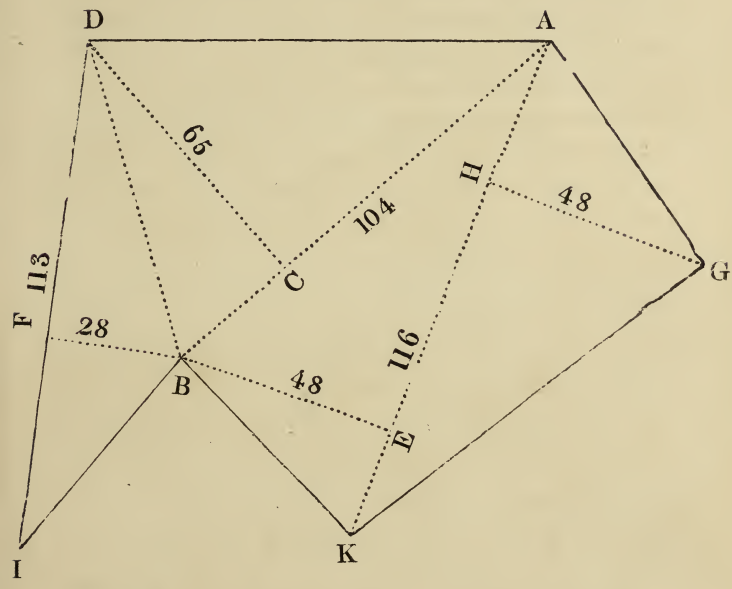
2) 250725

The half sum=125362·5

the area of the polygon.

Hence (125362·5÷100000) × 4=1 acre, 1·01448 roods, the content of the polygon B C E A H required.

Example 2. What is the content of the following field, the dimensions being yards?



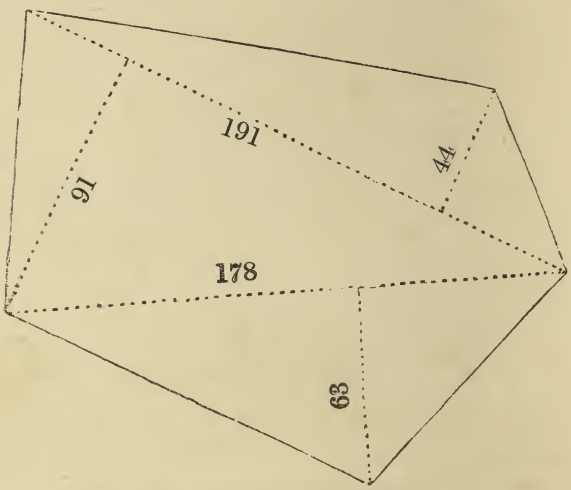
By Rules to Problem IV. *and* VII.

	Square Yards.
(H G + B E) × K A = (48 + 48) × 116	=11136
B A × C D = 104 × 65 - - - - - -	= 6760
I D × F B = 113 × 28 - - - - - -	= 3164
2) 21060	
10530	

Hence (10530 ÷ 4840) × 4 × 40 = 2 *a.* 0 *r.* $28\frac{12}{121}$ *p.*=*Answer.*

EXAMPLES FOR PRACTICE.

Example 1. The content of the following irregular polygon is required, the dimensions being yards:

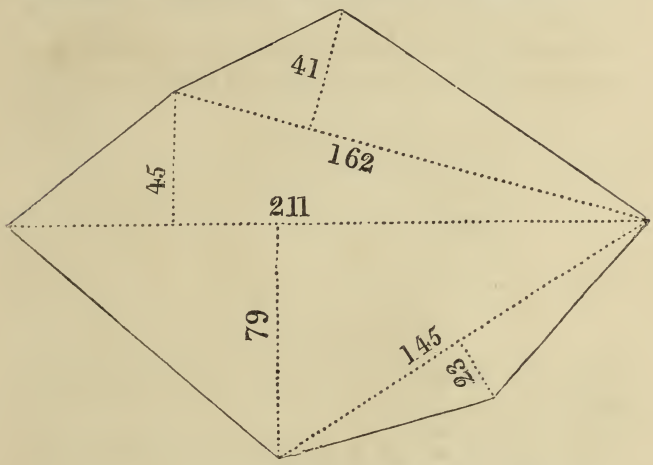


Content, 3 *a.* 3 *r.* $11\frac{67}{121}$ *p.*

Example 2. It is required to draw the plan and find the area of a five-sided field, A B C D E A *(by Rules to Prob.* IV., V., *and* VI.), whose dimensions are in chains, as follow: A B = 14, B C = 7, C D = 10, D E = 12, E A = 5, and the diagonal A C = 17 chains; also the angle at E being a right angle?

Ans. 14 *a.* 1·022701 *r.*

Example 3. The content of the following irregular polygon is required, the dimensions being yards :—



Content, 3 *a.* 2 *r.* $37\frac{45}{121}$ *p.*

Example 4. It is required to draw the plan and find the area of a five-sided field A B C D E A, *by Rule to Prob.* V., whose dimensions are as follow: A B = 2735 links, B C = 3115, C D = 2370, D E = 2925, and E A = 2220; also the diagonal from C to A = 3800, and from C to E = 4010 links, required the area.

Square Links.

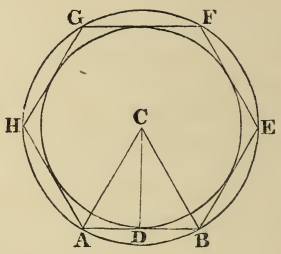
The area of the triangle A B C = 4204184
The area of the triangle A C E = 4137114
And the area of the triangle C D E = 3433161·06

The sum = 11774459·06 =

117 *a.* 2 *r.* 39·134496 *p.* = *Ans.*

PROBLEM X.

To determine the several particulars relating to regular Polygons up to 50 *sides.**



RULE I. *Given the side of a regular polygon to determine the area.*†

Multiply the square of the given side by the number in TABLE I., which is opposite the number of sides, and in the column of **AREAS**; the product will be the area required.

Example. Required the area of a regular polygon of seven sides, the length of each side being 15.

$15^2 \times 3{\cdot}6339126$ (tab. no.)$=817{\cdot}63034$ the area required.

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* The names and angles of the regular polygons are given, up to 50 sides, at *page* 46.

† Every regular polygon is composed of as many equal triangles as it has sides; consequently, if the area of one of these triangles be multiplied by the number of sides in the polygon, the product will give the area of the whole figure. But the area of each of the triangles is equal to the product of the perpendicular and half the base; therefore, the product of the perpendicular, and half the sum of the sides of the polygon, will give the area.

NOTE. *If the side, and perpendicular from the centre of the polygon to the middle of one of its sides, be known, (or the radius of the inscribed circle,)* the area can be found by multiplying half the sum of the sides by the perpendicular, or radius of the inscribed circle.

Example. The side of a regular polygon is 15, the number of sides seven, and the perpendicular, from the centre of the polygon to the middle of one of its sides, 15·573912, required the area.

$\frac{1}{2}(15\times7)\times15\cdot573912=817\cdot63038$, the area, *as before.*

RULE II. *Given the Radius of the circumscribed circle to determine the area.*

Multiply the square of the given Radius by the number in TABLE II., which is opposite the number of sides, and in the column of **AREAS**; the product will be the area required.

Example. If the Radius of the circumscribed circle be 17·2857375, and the number of sides of the regular polygon be seven, required the area.

$17\cdot2857375^2\times2\cdot7364103=817\cdot63042$ the area, *as before.*

RULE III. *Given the radius of the inscribed circle to determine the area.*

Multiply the square of the given radius by that number in TABLE III., which is opposite the number of sides, and in the column of **AREAS**; the product will be the area required.

Example. Let the radius of the inscribed circle be 15·573912, and the number of sides of the regular polygon be seven, required the area.

$15\cdot573912^2\times3\cdot3710222=817\cdot63042$ the area, *as before.*

RULE IV. *Given the side of a regular polygon to determine the Radius of the circumscribed circle.*

Multiply the given side by that number in TABLE IV., which is opposite the number of sides, and in the column of **RADII**; the product will be the length of the Radius required.

Example. The side of the regular polygon = 15, and the number of sides seven, required the Radius of the circumscribed circle.

$15 \times 1{\cdot}1523825 = 17{\cdot}28574$, the Radius of the circumscribed circle required.

RULE V. *Given the area of a regular polygon to determine the Radius of the circumscribed circle.*

Multiply the square root of the given area by the number in TABLE V., which is opposite the number of sides and in the column of **RADII**; the product will be the length of the Radius required.

Example. The area of a regular polygon = 817·63042, and the number of sides seven, required the Radius of the circumscribed circle.

$\sqrt{(817{\cdot}63042)} \times {\cdot}6045182 = 17{\cdot}28574$, the Radius, *as before.*

RULE VI. *Given the radius of the inscribed circle to determine the Radius of the circumscribed circle.*

Multiply the given radius by the number in TABLE VI., which is opposite the number of sides, and in the column of **RADII**; the product will be the length of the Radius required.

Example. Let the radius of the inscribed circle be 15·573912, and the number of sides of the regular polygon be seven, required the Radius of the circumscribed circle.

$15{\cdot}573912 \times 1{\cdot}1099163 = 17{\cdot}28574$, the Radius, *as before.*

RULE VII. *Given the side of a regular polygon to determine the radius of the inscribed circle.*

Multiply the given side by that number in TABLE VII., which is opposite the number of sides, and in the column of **radii**; the product will be the length of the radius required.

Example. If the side of a regular polygon be 15, and the number of its sides seven, required the radius of the inscribed circle.

$15 \times 1{\cdot}0382608 = 15{\cdot}573912$, the radius of the inscribed circle required.

RULE VIII. *Given the area of a regular polygon to determine the radius of the inscribed circle.*

Multiply the square root of the given area by that number in TABLE VIII., which is opposite the number of sides, and in the column of **radii**; the product will be the length of the radius required.

Example. The area of a regular polygon is 817·63042, the number of sides being seven, required the radius of the inscribed circle.

$\sqrt{(817{\cdot}63042)} \times {\cdot}5446521 = 15{\cdot}57391$, the radius, *as before.*

RULE IX. *Given the Radius of the circumscribed circle to determine the radius of the inscribed circle.*

Multiply the given Radius by that number in TABLE IX., which is opposite the number of sides, and in the column of **radii**; the product will be the length of the radius required.

Example. Let the Radius of the circumscribed circle = 17·28574, and the number of sides of the regular polygon be seven, required the radius of the inscribed circle.

$17{\cdot}28574 \times 9009628 = 15{\cdot}573912$, the radius, *as before.*

RULE X. *Given the area of the regular polygon to determine the side.*

Multiply the square root of the given area by that number in TABLE X., which is opposite the number of sides, and in the column of **SIDES**; the product will be the length of the side required.

Example. If the area of a regular polygon be 817·63042, the number of sides seven, required the length of one of its sides.

$\sqrt{(817{\cdot}63042)} \times {\cdot}5245812 = 15$, the length of the side required.

RULE XI. *Given the Radius of the circumscribed circle to find the side of the regular polygon.*

Multiply the given Radius by that number in TABLE XI., which is opposite the number of sides, and in the column of **SIDES**; the product will be the length of the side required.

Example. If the Radius of the circumscribed circle be 17·28574, required the length of the side of a regular polygon, whose number of sides is seven.

$17{\cdot}28574 \times {\cdot}8677676 = 15$, the length of the side, *as before.*

RULE XII. *Given the radius of the inscribed circle to determine the side of the polygon.*

Multiply the given radius by that number in TABLE XII., which is opposite the number of sides, and in the column of **SIDES**; the product will be the length of the required side.

Example. The radius of the inscribed circle is 15·573912, required the length of the side of a regular polygon, whose number of sides is seven.

$15{\cdot}573912 \times {\cdot}9631492 = 15$, the length of the side, *as before.*

TABLE I.

Areas of regular polygons up to 50 sides, the *side* of each being unity, computed from the formula

$$A = \frac{n}{4} \cot \frac{180^\circ}{n}$$

in which A = the area, and n = equal the number of sides.

No. of sides.	AREAS.	No. of sides.	AREAS.	No. of sides.	AREAS.	No. of sides.	AREAS.
3	·4330127	15	17·6423629	27	57·7499409	39	120·7755354
4	1·0000000	16	20·1093580	28	62·1268039	40	127·0620500
5	1·7204774	17	22·7355038	29	66·6627662	41	133·5082939
6	2·5980762	18	25·5207681	30	71·3577338	42	140·1130227
7	3·6339126	19	28·4652110	31	76·2121205	43	146·8771694
8	4·8284272	20	31·5687575	32	81·2255440	44	153·8007306
9	6·1818242	21	34·8315014	33	86·3980681	45	160·8824925
10	7·6942088	22	38·2533531	34	91·7298172	46	168·1245663
11	9·3656415	23	41·8344039	35	97·2207110	47	175·5253176
12	11·1961524	24	45·5745246	36	102·8704680	48	183·0846240
13	13·1857718	25	49·4738444	37	108·6797596	49	190·8045888
14	15·3345084	26	53·5323900	38	114·6481841	50	198·6818125

TABLE II.

Areas of regular polygons up to 50 sides, the *Radius of the circumscribing circle* being unity, computed from the formula

$$A = \frac{n}{2} \sin \frac{360^\circ}{n}$$

No. of sides.	AREAS.	No. of sides.	AREAS.	No. of sides.	AREAS.	No. of sides.	AREAS.
3	1·2990381	15	3·0505245	27	3·1133147	39	3·1280204
4	2·0000000	16	3·0614672	28	3·1152926	40	3·1286900
5	2·3776413	17	3·0705545	29	3·1170703	41	3·1293107
6	2·5980762	18	3·0781809	30	3·1186755	42	3·1298883
7	2·7364103	19	3·0846453	31	3·1201268	43	3·1304258
8	2·8284272	20	3·0901700	32	3·1214448	44	3·1309256
9	2·8925442	21	3·0949296	33	3·1226448	45	3·1313948
10	2·9389265	22	3·0990586	34	3·1237415	46	3·1318318
11	2·9735244	23	3·1026621	35	3·1247458	47	3·1322422
12	3·0000000	24	3·1058280	36	3·1256676	48	3·1326288
13	3·0207008	25	3·1086238	37	3·1265148	49	3·1329914
14	3·0371866	26	3·1111028	38	3·1272974	50	3·1333300

TABLE III.

Areas of regular polygons up to 50 sides, *the radius of the inscribed circle* being unity, computed from the formula

$$A = n \tan \frac{180^\circ}{n}$$

No.of sides.	AREAS.	No.of sides.	AREAS.	No.of sides.	AREAS.	No.of sides.	AREAS.
3	5·1961524	15	3·1883490	27	3·1558464	39	3·1484076
4	4·0000000	16	3·1825984	28	3·1548412	40	3·1480680
5	3·6327125	17	3·1778508	29	3·1539414	41	3·1477545
6	3·4641018	18	3·1738860	30	3·1531260	42	3·1474674
7	3·3710222	19	3·1705395	31	3·1523931	43	3·1471958
8	3·3137088	20	3·1676880	32	3·1517280	44	3·1469416
9	3·2757318	21	3·1652418	33	3·1511172	45	3·1467060
10	3·2491970	22	2·1631226	34	3·1505624	46	3·1464874
11	3·2298915	23	3·1612764	35	3·1500560	47	3·1462787
12	3·2153904	24	3·1596600	36	3·1495932	48	3·1460880
13	3·2042127	25	3·1582350	37	3·1491625	49	3·1459029
14	3·1954090	26	3·1569720	38	3·1487712	50	3·1457350

TABLE IV.

Radius of the circumscribing circle of every regular polygon up to 50 sides, the *side* of each being unity, computed from the formula

$$R = \tfrac{1}{2} \operatorname{cosec} \frac{180^\circ}{n}$$

in which R = the Radius of the circumscribing circle.

No.of sides.	RADII.	No.of sides.	RADII.	No.of sides.	RADII.	No.of sides.	RADII.
3	·5773503	15	2·4048672	27	4·3068951	39	6·2137666
4	·7071068	16	2·5629155	28	4·4657083	40	6·3727475
5	·8506508	17	2·7210970	29	4·6245414	41	6·5317653
6	1·0000000	18	2·8793853	30	4·7833861	42	6·6907572
7	1·1523825	19	3·0377692	31	4·9422682	43	6·8497696
8	1·3065630	20	3·1962266	32	5·1011600	44	7·0087998
9	1·4619022	21	3·3547557	33	5·2600645	45	7·1677935
10	1·6180340	22	3·5133383	34	5·4189880	46	7·3268444
11	1·7747331	23	3·6719752	35	5·5779245	47	7·4858794
12	1·9318517	24	3·8306488	36	5·7368565	48	7·6448940
13	2·0892913	25	3·9893649	37	5·8958213	49	7·8039765
14	2·2469806	26	4·1481206	38	6·0547953	50	7·9629855

TABLE V.

Radius of the circumscribing circle of every regular polygon up to 50 sides, the *area* of each being unity, computed from the formula

$$R = \sqrt{\left(\frac{2}{n} \operatorname{cosec} \frac{360^\circ}{n}\right)}$$

No. of sides.	RADII.	No. of sides.	RADII.	No. of sides.	RADII.	No. of sides.	RADII.
3	·8773826	15	·5725491	27	·5667462	39	·5654124
4	·7071068	16	·5715249	28	·5665662	40	·5653518
5	·6485252	17	·5706787	29	·5664044	41	·5652959
6	·6204032	18	·5699712	30	·5662587	42	·5652438
7	·6045182	19	·5693737	31	·5661271	43	·5651953
8	·5946036	20	·5688645	32	·5660074	44	·5651499
9	·5879765	21	·5684270	33	·5658988	45	·5651075
10	·5833184	22	·5680482	34	·5657996	46	·5650683
11	·5799149	23	·5677181	35	·5657085	47	·5650313
12	·5773509	24	·5674287	36	·5656250	48	·5649962
13	·5753686	25	·5671735	37	·5655485	49	·5649638
14	·5738050	26	·5669475	38	·5654778	50	·5649329

TABLE VI.

Radius of the circumscribing circle of every regular polygon up to 50 sides, the *radius of the inscribed circle* being unity, computed from the formula

$$R = \sec \frac{180^\circ}{n}$$

No. of sides.	RADII.	No. of sides.	RADII.	No. of sides.	RADII.	No. of sides.	RADII.
3	2·0000000	15	1·0223406	27	1·0068077	39	1·0032533
4	1·4142136	16	1·0195912	28	1·0063276	40	1·0030922
5	1·2360680	17	1·0173219	29	1·0058966	41	1·0029429
6	1·1547005	18	1·0154266	30	1·0055083	42	1·0028040
7	1·1099163	19	1·0138273	31	1·0051571	43	1·0026749
8	1·0823922	20	1·0124651	32	1·0048386	44	1·0025544
9	1·0641778	21	1·0112954	33	1·0045487	45	1·0024419
10	1·0514622	22	1·0102833	34	1·0042841	46	1·0023367
11	1·0422171	23	1·0094016	35	1·0040420	47	1·0022382
12	1·0352762	24	1·0086290	36	1·0038198	48	1·0021457
13	1·0299279	25	1·0079480	37	1·0036155	49	1·0020588
14	1·0257169	26	1·0073447	38	1·0034272	50	1·0019772

TABLE VII.

Radius of the inscribed circle of every regular polygon up to 50 sides, the *side* of each being unity, computed from the formula

$$r = \tfrac{1}{2} \cot \frac{180^\circ}{n}$$

in which r = the radius of the inscribed circle.

No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.
3	·2886752	15	2·3523151	27	4·2777734	39	6·1936172
4	·5000000	16	2·5136698	28	4·4376289	40	6·3531025
5	·6881910	17	2·6747652	29	4·5974322	41	6·5125997
6	·8660254	18	2·8356409	30	4·7571823	42	6·6720487
7	1·0382608	19	2·9963380	31	4·9169110	43	6·8314963
8	1·2071068	20	3·1568758	32	5·0765965	44	6·9909423
9	1·3737387	21	3·3172859	33	5·2362466	45	7·1503330
10	1·5388418	22	3·4775776	34	5·3958716	46	7·3097638
11	1·7028439	23	3·6377743	35	5·5554692	47	7·4691625
12	1·8660254	24	3·7978771	36	5·7150260	48	7·6285260
13	2·0285803	25	3·9579076	37	5·8745816	49	7·7879424
14	2·1906441	26	4·1178762	38	6·0341150	50	7·9472725

TABLE VIII.

Radius of the inscribed circle of every regular polygon up to 50 sides, the *area* of each being unity, computed from the formula

$$r = \sqrt{\left(\frac{1}{n} \cot \frac{180^\circ}{n}\right)}$$

No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.
3	·4386914	15	·5600375	27	·5629139	39	·5635791
4	·5000000	16	·5605432	28	·5630040	40	·5636090
5	·5246679	17	·5609619	29	·5630846	41	·5636380
6	·5372850	18	·5613120	30	·5631567	42	·5636636
7	·5446521	19	·5616084	31	·5632229	43	·5636879
8	·5493421	20	·5618608	32	·5632826	44	·5637110
9	·5525171	21	·5620783	33	·5633367	45	·5637309
10	·5547687	22	·5622663	34	·5633864	46	·5637516
11	·5564243	23	·5624306	35	·5634319	47	·5637701
12	·5576775	24	·5625742	36	·5634727	48	·5637865
13	·5586495	25	·5627012	37	·5635113	49	·5638042
14	·5594185	26	·5628141	38	·5635468	50	·5638181

TABLE IX.

Radius of the inscribed circle of every regular polygon up to 50 sides, the *Radius of the circumscribed circle* being unity, computed from the formula

$$r = \cos \frac{180^\circ}{n}$$

No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.	No.of sides.	radii.
3	·5000000	15	·9781476	27	·9932384	39	·9967573
4	·7071068	16	·9807853	28	·9937122	40	·9969173
5	·8090170	17	·9829731	29	·9941380	41	·9970658
6	·8660254	18	·9848078	30	·9945219	42	·9972038
7	·9009688	19	·9863613	31	·9948693	43	·9973322
8	·9238795	20	·9876883	32	·9951847	44	·9974521
9	·9396926	21	·9888308	33	·9954719	45	·9975641
10	·9510565	22	·9898215	34	·9957342	46	·9976688
11	·9594930	23	·9906859	35	·9959743	47	·9977668
12	·9659258	24	·9914449	36	·9961947	48	·9978589
13	·9709418	25	·9921147	37	·9963975	49	·9979454
14	·9749279	26	·9927089	38	·9965844	50	·9980267

TABLE X.

Side of every regular polygon up to 50 sides, the *area* of each being unity, computed from the formula

$$s = 2\sqrt{\left(\frac{1}{n} \tan \frac{180^\circ}{n}\right)}$$

in which s = the side.

No.of sides.	SIDES.	No.of sides.	SIDES.	No.of sides.	SIDES.	No.of sides.	SIDES.
3	1·5196714	15	·2380793	27	·1315903	39	·0909936
4	1·0000000	16	·2229981	28	·1268705	40	·0887139
5	·7623870	17	·2097238	29	·1224783	41	·0865459
6	·6204032	18	·1979489	30	·1183804	42	·0844815
7	·5245812	19	·1874317	31	·1145483	43	·0825133
8	·4550898	20	·1779801	32	·1109569	44	·0806347
9	·4021996	21	·1694393	33	·1075842	45	·0788398
10	·3605106	22	·1616834	34	·1044107	46	·0771233
11	·3267618	23	·1546085	35	·1014193	47	·0754798
12	·2988585	24	·1481286	36	·0985949	48	·0739051
13	·2753895	25	·1421715	37	·0959237	49	·0723947
14	·2553672	26	·1366750	38	·0933936	50	·0709448

TABLE XI.

Side of every regular polygon up to 50 sides, the *Radius of the circumscribed circle* being unity, computed from the formula

$$s = 2 \sin \frac{180^\circ}{n}$$

No. of sides.	SIDES.	No. of sides.	SIDES.	No. of sides.	SIDES.	No. of sides.	SIDES.
3	1·7320508	15	·4158234	27	·2321858	39	·1609332
4	1·4142136	16	·3901806	28	·2239290	40	·1569182
5	1·1755706	17	·3674990	29	·2162380	41	·1530984
6	1·0000000	18	·3472964	30	·2090570	42	·1494602
7	·8677676	19	·3291892	31	·2023366	43	·1459906
8	·7653668	20	·3128690	32	·1960344	44	·1426784
9	·6840402	21	·2980846	33	·1901122	45	·1395130
10	·6180340	22	·2846296	34	·1845368	46	·1364848
11	·5634652	23	·2723332	35	·1792786	47	·1335852
12	·5176380	24	·2610524	36	·1743114	48	·1308062
13	·4786312	25	·2506664	37	·1696120	49	·1281404
14	·4450418	26	·2410734	38	·1651586	50	·1255810

TABLE XII.

Side of every regular polygon up to 50 sides, the *radius of the inscribed circle* being unity, computed from the formula

$$s = 2 \tan \frac{180^\circ}{n}$$

No. of sides.	SIDES.	No. of sides.	SIDES.	No. of sides.	SIDES.	No. of sides.	SIDES.
3	3·4641016	15	·4251132	27	·2337664	39	·1614568
4	2·0000000	16	·3978248	28	·2253458	40	·1574034
5	1·4530850	17	·3738643	29	·2175132	41	·1535490
6	1·1547006	18	·3526540	30	·2102084	42	·1498794
7	·9631492	19	·3337410	31	·2033802	43	·1463812
8	·8284272	20	·3167688	32	·1969830	44	·1430428
9	·7279404	21	·3014516	33	·1909768	45	·1398536
10	·6498394	22	·2875566	34	·1853272	46	·1368038
11	·5872530	23	·2748936	35	·1800032	47	·1338842
12	·5358984	24	·2633050	36	·1749774	48	·1310870
13	·4929558	25	·2526588	37	·1702250	49	·1284042
14	·4564870	26	·2428440	38	·1657248	50	·1258294

EXAMPLES FOR PRACTICE.

Example 1. The side of a regular hexagon A B = 25 yards, and the perpendicular, from the centre to the middle of one of its sides, C D = 21·650635 yards, required the area. *Ans.* 1623·797625.

Example 2. Required the area of a nonagon, whose side measures 10 chains, and perpendicular 13·73 chains. *Ans.* = 61 *a.* 3 *r.* 5·6 *p.*

Example 3. Required the area of a trigon (an equilateral triangle), whose side is 20, and perpendicular 5·773502. *Ans.* = 173·20506.

Example 4. The side of a regular hexagon is A B = 25 yards; required the radii of the inscribed and circumscribed circle.

Ans. { 21·650635 yards, inscribed radius.
25·000000 yards, circumscribed Radius.

Example 5. The length of the side of a regular triacontagon being $13\frac{1}{2}$ yards; required the area, and also the radii of the circle which circumscribes and inscribes it.

Ans. { The area of the triacontagon = 13004·94699 *sq.y.*
Radius of circumscribed circle = 64·57571 *yds.*
Radius of inscribed circle - = 64·22196 *yds.*

Example 6. The Radius of a circle which circumscribes a regular octadecagon being 60 yards; required the area, the radius of the inscribed circle, and length of each side.

Ans. { The area of the octadecagon = 11081·45124 *sq.y.*
Radius of inscribed circle = 59.08847 *yds.*
The length of each side - = 20·83778 *yds.*

Example 7. The radius of a circle which inscribes a regular pentecontagon being 54 yards; required the area, the Radius of the circumscribed circle, and length of each side.

Ans. { The area of the pentecontagon = 9172·96326 *sq.y.*
Radius of circumscribed circle = 54·10677 *yds.*
The length of each side - = 6·79479 *yds.*

Example 8. The area of a regular eicosagon being 888949·151281 square links; required the radii of the circle which circumscribes and inscribes it, and also the length of each side.

Ans. { Radius of circumscribed circle = 536·34877 *links.*
Radius of inscribed circle - = 529·74540 *links.*
The length of each side - = 167·80694 *links.*

Example 9. A regular polygon of 16 sides having been projected (by Mr. Leroux, architect,) in the centre of Clarendon-square, Sommer's Town, London, for the erection of thirty-two houses of certain dimensions, the fronts of the houses facing the sides of the polygon; and it is found by actual measurement, that the Radius of the circumscribed circle is = 162½ feet; required to find the following dimensions of the hexadecagon: the area, the length of each side, and the length of the radius of the inscribed circle.

Ans. { Area of the hexadecagon = 80841·86825 *sq. ft.*
= 1 *a.* 3 *r.* 16·940 *p.*
Radius of inscribed circle = 159·3776 *feet*, or the perpendicular from the centre of the polygon to the middle of one of its sides.
The length of each side = 63·40435 *feet.*

PROBLEM XI.

Given the Diameter of a Circle to find the Circumference, or given the Circumference to find the Diameter.

CASE I.

When the Diameter is given, to find the Circumference.

RULE I. As 7 is to 22, so is the diameter given to the circumference.

RULE II. As 1 is to 3·1416, so is the diameter given to the circumference.

RULE III. As 113 is to 355, so is the diameter given to the circumference.

Example. The diameter of a circle is 22·6, what is the circumference.

By Rule I. $(22{\cdot}6 \times 22) \div 7 = 71{\cdot}028$, the circumference required *nearly.*

By Rule II. $22{\cdot}6 \times 3{\cdot}1416 = 71{\cdot}00016$, the circumference *more nearly.*

By Rule III. $(22{\cdot}6 \times 355) \div 113 = 71$, the circumference *still nearer.*

CASE II.

When the Circumference is given, to find the Diameter.

RULE I. As 22 is to 7, so is the circumference given to the diameter.

RULE II. As 3·1416 is to 1, so is the circumference given to the diameter; or, which is the same

thing,—As 1 is to ·318309*, so is the circumference given to the diameter.

RULE III. As 355 is to 113, so is the circumference given to the diameter.

Example. The circumference of a circle is 71, what is the diameter?

By Rule I. $(71 \times 7) \div 22 = 22{\cdot}5909$, the diameter required *nearly.*

By Rule II. $71 \times {\cdot}318309 = 22{\cdot}599939$, the diameter *more nearly.*

By Rule III. $(71 \times 113) \div 355 = 22{\cdot}6$, the diameter *still nearer.*

EXAMPLES FOR PRACTICE.

Example 1. If the diameter of the earth be 7958 miles, what is its circumference, supposing it a perfect sphere? *Ans.* 25000·8528 miles.

Example 2. Required to find the diameter of the globe of the earth, supposing its circumference to be 25000 miles? *Ans.* 7957$\frac{3}{4}$ miles, *nearly.*

Example 3. Supposing the diameter of the sun to be 883220 miles; what is its circumference?

Ans. 2774723·952 miles.

Example 4. The circumference of the moon is supposed to be 6850 miles; what is its diameter?

Ans. 2180·41762 miles.

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* This number is found by dividing 1 by 3·1416.

PROBLEM XII.

To find the Area of a Circle.

RULE I. *When the diameter and circumference are given.*—Multiply half the circumference by half the diameter, for the area.

Or thus,

Multiply the whole circumference by the whole diameter, and take $\frac{1}{4}$ of the product for the area.

RULE II. *When the diameter only is given.*—Multiply the square of the diameter by ·7854, or the square of the radius by 3·1416, and the product of either will be the area.

RULE III. *When the diameter is given.*—As 452 is to 355, so is the square of the diameter to the area.

RULE IV. *When the diameter is given.*—As 14 is to 11, so is the square of the diameter to the area.

RULE V. *When the circumference is given.*—Multiply the square of the circumference by ·07958, and the product will be the area.

RULE VI. *When the circumference is given.*—As 88 is to 7, so is the square of the circumference to the area.

RULE VII. *When the circumference is given.*—As

1420 is to 113, so is the square of the circumference to the area.

Example. Required the area of a circle, (by the seven Rules given to this Problem) whose diameter is 22·6 chains, and circumference 71 chains.

By Rule I. $(71 \div 2) \times (22{\cdot}6 \div 2) = 401{\cdot}15$ chains = 1*st Ans.*
By Rule II. $22{\cdot}6^2 \times {\cdot}7854 = 401{\cdot}150904$ chains = 2*d Ans.*
By Rule III. $(22{\cdot}6^2 \times 355) \div 452 = 401\ 15$ chains = 3*rd Ans.*
By Rule IV. $(22{\cdot}6^2 \times 11) \div 14 = 401{\cdot}31$ chains = 4*th Ans.*
By Rule V. $71^2 \times {\cdot}07958 = 401\ 16278$ chains = 5*th Ans.*
By Rule VI. $(71^2 \times 7) \div 88 = 400{\cdot}9886$ chains = 6*th Ans.*
By Rule VII. $(71^2 \times 113) \div 1420 = 401{\cdot}15$ chains = 7*th Ans.*

Note 1. *To find the area of a circular ring, or the space included between two concentric circles.*—Rule I. Multiply the sum of the diameters of the two circles by their difference, and that product by ·7854; the last product will give the area of the ring. *Or thus:* Take the difference of the areas of the two circles for the area of the ring. —Rule II. Multiply the sum of the circumferences by their difference, and the product by ·07958; this last product will give the area of the ring.

Example. The diameters of two concentric circles being 10 and 6, required the area of the ring contained between these two circles.

By Rule I. $(10 + 6) \times (10 - 6) \times {\cdot}7854 = 16 \times 4 \times {\cdot}7854 = 50{\cdot}2656$, the area of the ring, or the area of the space contained between the two circumferences.

Or thus,

Here $10^2 \times {\cdot}7854 = 78{\cdot}54$, the area of the greater circle.
Again, $6^2 \times {\cdot}7854 = 28{\cdot}2744$, the area of the less circle.

The difference $= 50{\cdot}2656$, the area of the ring, *as before.*

By Rule II. Taking the same example as above, we shall find the circumferences to be 31·4160, and 18·8496.

Hence $(31{\cdot}4160 + 18{\cdot}8496) \times (31{\cdot}4160 - 18{\cdot}8496) \times {\cdot}07958 = 50{\cdot}2656 \times 12{\cdot}5664 \times {\cdot}07958 = 50{\cdot}26731466$, the area of the ring, *as before.*

Note 2. *When the area of the circle is given, to find the diameter.*—Multiply the square root of the given area by 1·12838, the product will be the diameter; or divide the given area by ·7854, the square root of the quotient will be the diameter.

Note 3. *When the area of the circle is given, to find the circumference.*—Multiply the square root of the given area by 3·5449, the product will be the circumference; or divide the given area by ·0795773, the square root of the quotient will be the circumference.

Note 4. To find the area of a semicircle, or of a quadrant, find the area of the whole circle, and divide it by 2 for the semicircle, and by 4 for the quadrant.

Note 5. When the *diameter* of a circle is 1, the *circumference* is $= 3{\cdot}14159265358979323846 +$; and the *area* is $=$ $·78539816339744830961 +$; and when the *circumference* is 1, the *area* is $=$ $·07957747154594766 7+$; also the *reciprocal* of 3·14159, &c. is $=$ $·31830988618379067 1+$. To these numbers 3·1416, ·7854, ·0795775, and ·31831, respectively, are approximations sufficiently near the truth for all common purposes. But when any calculation occurs in which greater exactness is necessary, the learner has only to use more decimal places according to the degree of accuracy required.

The following Rules relative to the Circle will be found very useful:—

Rule I. *When the diameter is given, to find the side of the square, equal in area to the circle.*—The diameter of any circle, multiplied by ·8862269, will give the side of a square equal in area.

Rule II. *When the circumference is given, to find the side of the square, equal in area to the circle.*—The circumference of any circle, multiplied by ·2820948, will give the side of a square equal in area.

Rule III. *When the diameter is given, to find the side of the inscribed square.*—The diameter of any circle, multiplied by ·7071068, will give the side of a square inscribed in that circle.

Rule IV. *When the circumference is given, to find the side of the inscribed square.*—The circumference of any circle, multiplied by ·2250791, will give the side of a square inscribed in that circle.

RULE V. *When the area is given, to find the side of the inscribed square.*—The area of any circle, multiplied by ·6366198, and the square root of the product extracted, will give the side of a square inscribed in that circle.

RULE VI. *When the side of a square is given, to find the diameter of its circumscribing circle.*—The side of any square, multiplied by 1·4142136, will give the diameter of its circumscribing circle.

RULE VII. *When the side of a square is given, to find the circumference of its circumscribing circle.*—The side of any square, multiplied by 4·4428829, will give the circumference of its circumscribing circle.

RULE VIII. *When the side of a square is given, to find the diameter of a circle equal in area to it.*—The side of any square, multiplied by 1·1283791, will give the diameter of a circle equal in area to the square.

RULE IX. *When the side of a square is given, to find the circumference of a circle equal in area to it.*—The side of any square, multiplied by 3·5449076, will give the circumference of a circle equal in area to the square.

EXAMPLES FOR PRACTICE.

Example 1. What is the area of a circle, whose diameter is 1, and circumference 3·1416?

Ans. ·7854.

Example 2. How many square feet does a circle contain, the circumference being 10·9956 yards?

Ans. 86·55571386 sq. feet.

Example 3. The diameter of a circle is ·318309, and the circumference being 1, required the area of the quadrant? *Ans.* ·019894.

Example 4. The area of a circular plantation is known to occupy an acre and a half of ground; what must be the length of a line which will strike the circle? *Ans.* 48·072 yards.

Example 5. Find the circumference of a circle, whose area is 4840 square yards?

Ans. 246·619 yards.

Example 6. Required the diameter and circumference of a circle, whose area is 5760 square yards.

Ans. { Diameter - - = 85·638 yards.
Circumference = 269·039 yards.

Example 7. Required the area of a semicircle, and of a quadrant, the diameter of the circle being 10 yards.

Ans. { The area of the whole circle = 78·54 sq. yds.
The semicircle - - = 39·27 yards.
The quadrant - - = 19·635 yards.

Example 8. The length of a line, with which my gardener formed a circular fish-pond, was exactly $27\frac{3}{4}$ yards; pray what quantity of ground did the fish-pond take up?

Ans. 2419·22835 sq. yds.; or $\frac{1}{2}$ an acre, *nearly*.

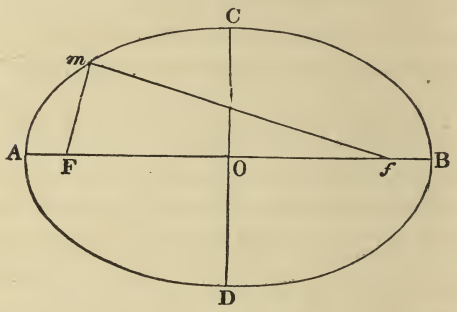
Example 9. The surveying-wheel is so contrived as to turn just twice in the length of a pole, or $16\frac{1}{2}$ feet: in going round a circular bowling-green, I observed it to turn exactly 200 times; what is the area of the bowling-green?

Ans. 4 *a.* 3 *r.* 35·8 *p.*

Example 10. The inner diameter of a circular light-house = 25 feet, and the thickness of the wall = 3·5 feet; how many square feet of ground does the wall stand upon? *Ans.* 313·3746 square feet.

PROBLEM XIII.

To find the Area, and also the Circumference of an Ellipse, by having the transverse and conjugate Diameters given.*



First, to find the area of an ellipse.

RULE.—Multiply the product of the two diameters by ·7854, for the area of the ellipse.

Example. The transverse diameter of an ellipse A B = 50 yards, and the conjugate diameter C D = 40 yards; required the area.

Here A B × B C × ·7854 = 50 × 40 × ·7854 = 1570·8 square yards, the area of the ellipse required.

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* For further information on some general properties of the Ellipse, see page 162.

Second, to find the circumference of an ellipse.

RULE. Multiply the square root of half the sum of the squares of the two axes by 3·1416, and the product will be the circumference *nearly*.

Example. What is the circumference of an ellipse, of which the transverse and conjugate axes A B and C D = 24 yards and 18 yards?

Here $\surd[\frac{1}{2}(AB^2+CD^2)] \times 3{\cdot}1416 = \surd[\frac{1}{2}(24^2+18^2)] \times 3{\cdot}1416 = 21{\cdot}2132 \times 3{\cdot}1416 = 66{\cdot}643$ yards, the circumference of the ellipse A C B D A required.

Note. To find the area of an elliptical ring, or the space included between the circumferences of two concentric and similar ellipses.—RULE. From the product of the two diameters of the greater ellipse, subtract the product of the two diameters of the less; the remainder, multiplied by ·7854, will be the area of the ring.—*Or thus:* Find the area of the two ellipses, and subtract the less from the greater; the remainder will be the area of the ring.

Example. Required the area of the space contained between the two concentric and similar ellipses, the transverse diameter of the greater ellipse A B=70, and the conjugate diameter C D=56; also another ellipse within this, having the same centre, whose transverse and conjugate diameters are 50 and 36.

Here $(70 \times 56) - (50 \times 36) \times {\cdot}7854 = (3920-1800) \times {\cdot}7854 = 2120 \times {\cdot}7854 = 1665{\cdot}048$, the area of the ring, or the area of the space contained between the two circumferences.

Or thus,

Here $70 \times 50 \times {\cdot}7854 = 3078{\cdot}768$, the area of the greater ellipse.
Again, $50 \times 36 \times {\cdot}7854 = 1413{\cdot}720$, the area of the less ellipse.

The difference $= 1665{\cdot}048$, the area of the ring, *as before*.

EXAMPLES FOR PRACTICE.

Example 1. What is the periphery of the ellipse A C B D A, whose two axes A B and C D = 140 and 120 yards? *Ans.* 409·6 yards.

Example 2. The transverse diameter A B = 61·6 yards, and the conjugate diameter C D = 44 yards; required the area of the ellipse A C B D A.

Ans. 2128·74816 square yards.

Example 3. Required the circumference of an ellipse A C B D A, whose transverse and conjugate diameters A B and C D = 210 and 180 yards. *Ans.* 614·4 yards.

Example 4. The two axes of an ellipse are A B = 35 yards, and C D = 25 yards; required the area of the ellipse A C B D A.

Ans. 687·225 square yards.

Example 5. The diameters of an elliptical piece of ground are 330 and 220 feet; how many quicks will plant the fence forming the circumference, supposing them to be set five inches asunder?

Ans. 2073.

Example 6. The transverse diameter of an ellipse = 76 yards, the conjugate diameter = 56 yards; and the transverse diameter of another ellipse, having the same centre = 50 yards, and the conjugate diameter = 30 yards; required the area of the space contained between their circumferences.

Ans. 2164·5624 square yards.

PART THE THIRD.

TO SURVEY WITH THE CHAIN AND CROSS.

DESCRIPTION OF INSTRUMENTS.

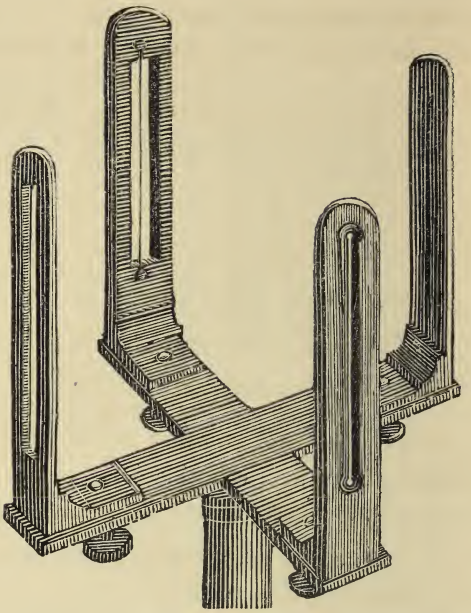
The Surveying-Chain contains one hundred links, and is twenty-two yards, or four poles, in length.

The Surveying-Arrows should be about two feet long, of thick wire, fixed in wood, in handles painted red.

The Offset-Staff is ten links long, divided into links.

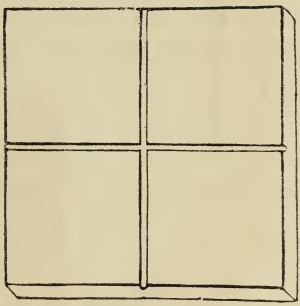
Other staffs about six feet long, for marks.

The Cross consists of two pair of sights fixed at right angles to each other.

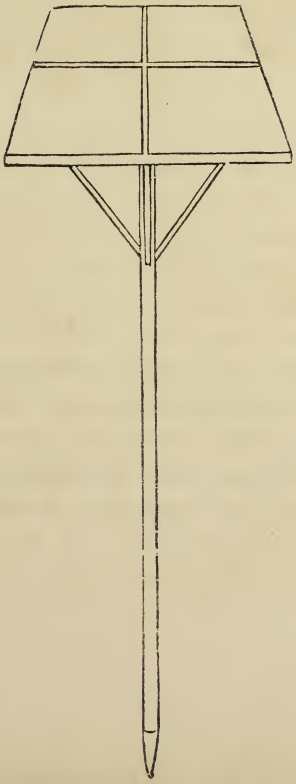


This, when in use, is screwed on a staff, about $4\frac{1}{2}$ feet long, having a sharp point at the bottom, to stick in the ground; the four sights screw off, to make the instrument convenient for the pocket; and the staff unscrews into three parts, to go into a portmanteau. The accuracy of this instrument depends on the sights being exactly at right angles to each other. It may be proved by looking at one object through two of the sights, and observing, at the same time, without moving the instrument, another object through the other two sights; then, turning the cross upon the staff, look at the same objects through the opposite sights; if they are accurately in the direction of the sights, it is correct.

An inferior instrument may be made of a square piece of wood, with two grooves cut in it at right angles :—



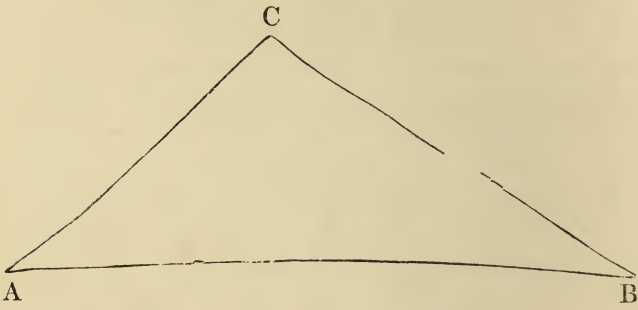
This must be fastened on a staff of the same length as the other :—



The use of the cross is to raise a perpendicular on a given line to a point out of the line. This may be done with much less trouble by laying the offset staff perpendicular to the chain as it lies extended on the ground, and moving it backward or forward till it is in a line with the given point: if the distance be measured from that part of the chain which the offset-staff touches, it will vary very little from a true perpendicular, and be sufficiently accurate: this may be easily proved by measuring a perpendicular both ways.

To survey the Triangle A B C.

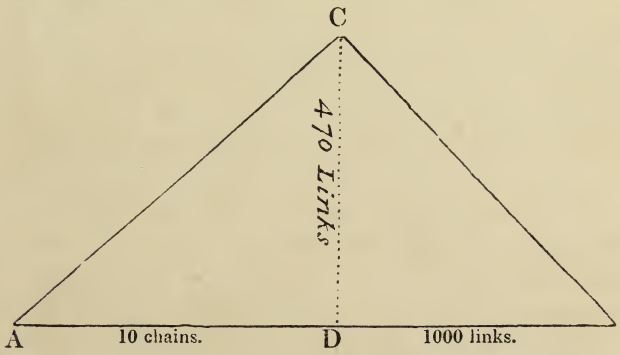
Draw a rough sketch in the field-book, thus:—



Set up marks at the angles A B C, and begin to measure the longest line at A; let the person who leads the chain take the arrows, and when he has proceeded to the length of it, the follower must direct him to get in a straight line with the mark at B;

he must there put down an arrow, proceed to the length of another chain, and put down another arrow (the follower must take the arrows up as he comes at them, and by that means keep an account of the line he is measuring); proceed thus, till you think you are nearly opposite the mark C; fix your cross at D in such way as to see the marks A and B through one of the sights, when, if you are in the true place for the perpendicular, you will see through the other mark C; if you cannot see it, you must move backwards or forwards till you are in a position to see the three marks without moving the cross; leave a mark at D, and finish measuring the base, allow it to be 10 chains; then measure the perpendicular, allow it to be 4 chains, 70 links.

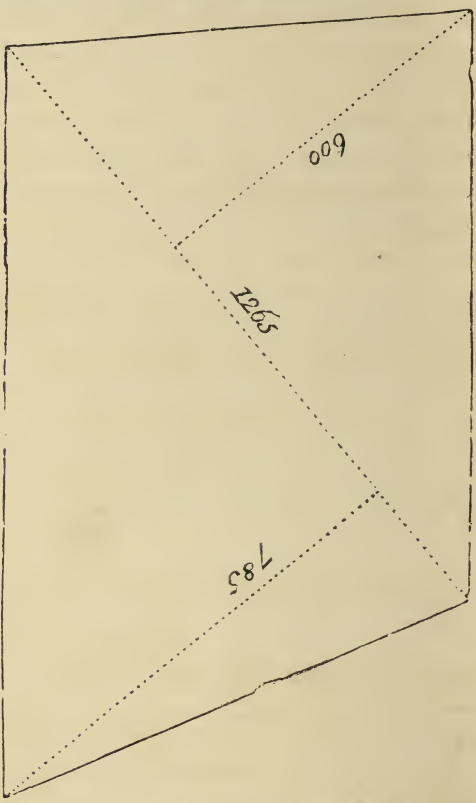
Note.—It is best to make use of the number of links instead of chains, they being as easily expressed, and more convenient to work with, therefore the following fields are all measured in links:—



Content, 2 *a.* 1 *r.* 16 *p.*

To survey a Trapezium.

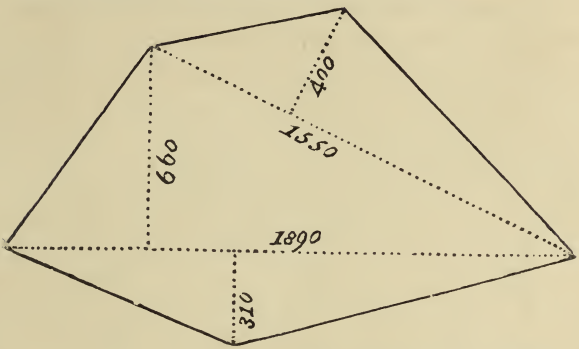
Measure a diagonal across, which will divide it into two triangles, and measure a perpendicular to each triangle, as in the following diagram:—



Content, 8 *a.* 3 *r.* 162 *p.*

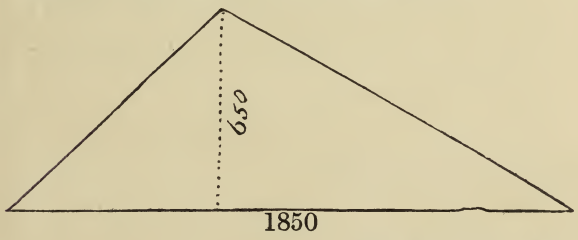
All other fields may be surveyed by dividing them into triangles, and their contents found as in the last two examples.

The content of the following field is required:



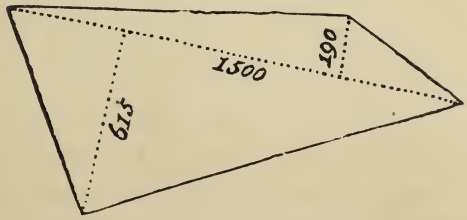
Ans. 12 *a.* 1 *r.* 2·64 *p.*

What is the content of the following field?



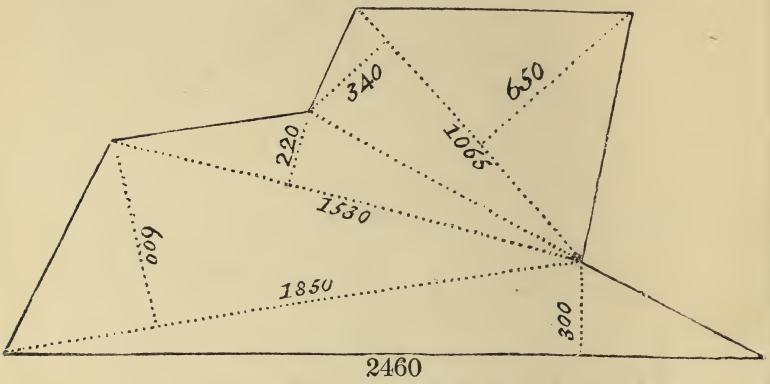
Ans. 6 *a.* 0 *r.* 2 *p.*

What is the content of the following field?



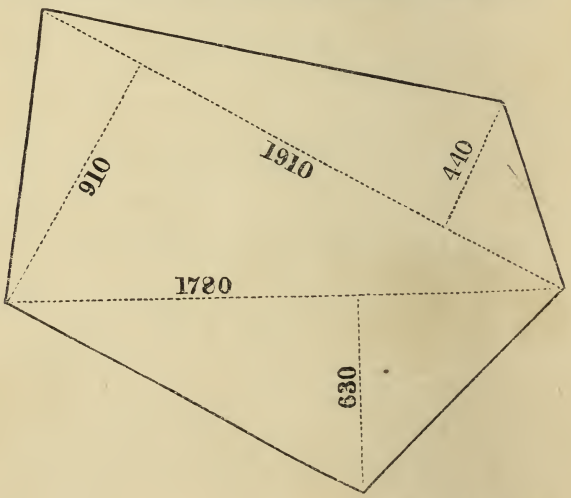
Ans. 6 *a.* 0 *r.* 6 *p.*

Required the content of the following field:—



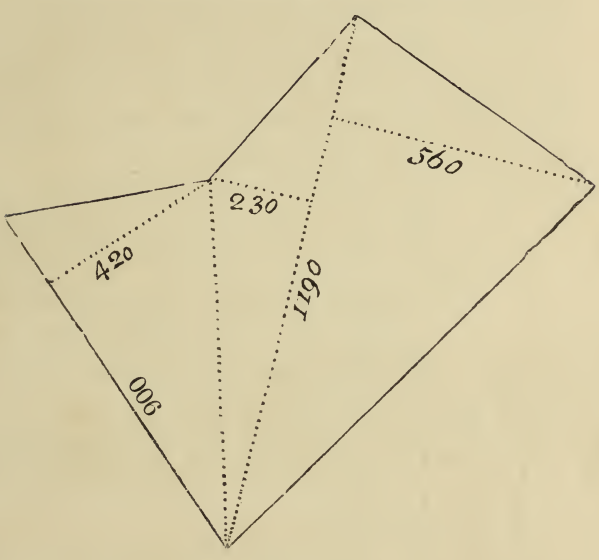
Ans. 16 *a.* 0 *r.* 31·16 *p.*

The content of the following field is required:—

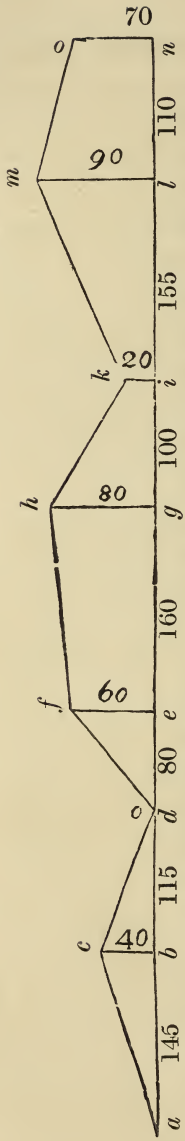


Ans. 18 *a.* 2 *r.*

What is the content of the following field?



Ans. 6 *a.* 2 *r.* 14·48 *p.*



When a boundary is not straight, a straight line must be measured as near to it as can be, and offsets measured in a perpendicular direction from the chain-line to the boundary, at every angle or corner.

To raise a perpendicular for offsets, let the chain lie extended on the ground, and lay the offset-pole perpendicular to it, and pointing to the angle of the offset, observe at what length on the chain the perpendicular is raised, &c.

The offsets will form right-angled triangles, as *a b c*, or trapezoids, as *e f g h*.

Rule *for computing the contents of offsets.*—For a triangle, multiply the perpendicular and base together; for a trapezoid, multiply the sum of the two perpendiculars by the base, add all the products together, and divide by 2, the quotient will be the content.—*See Prob.* IV. *and* VIII. *Pages* 66 & 78.

Note.—When one perpendicular belongs to two triangles, as *b c*, treat them as one triangle, and multiply *a d* into *b c*.—*See Prob.* IV. *Page* 66.

In measuring a line which has offsets, note down, in the rough sketch, the distance of each offset, from that end of the line you began to measure from.

The length of each base may then be found, by subtracting the first distance from the second, the second from the third, &c. This will be easily understood, by inspecting the annexed figure and following work:—

By Rules to Problems IV. *and* VIII. *Pages* 66 *&* 78.

Square Links.

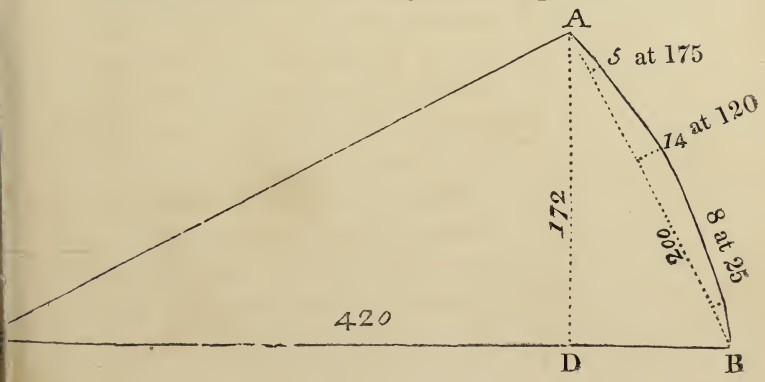
$ad \times cb = 260 \times 40 =$ - - - 10400
$de \times ef = 80 \times 60 =$ - - - 4800
$(ef + gh) \times eg = (60 + 80) \times 160 = 22400$
$gh + ik) \times gi = (80 + 20) \times 100 = 10000$
$(ik + lm) \times il = (20 + 90) \times 155 = 17050$
$(lm + no) \times ln = (90 + 70) \times 100 = 17600$

2) 82250

41125 square links,

the content of the offsets from *a* to *n*.

The content of the following field is required:—



By Rules to Problems IV. *and* VIII. *Pages* 66 *&* 78.

C B × A D = 420 × 172 = 72240

Double the area of the off-sets from B to A.

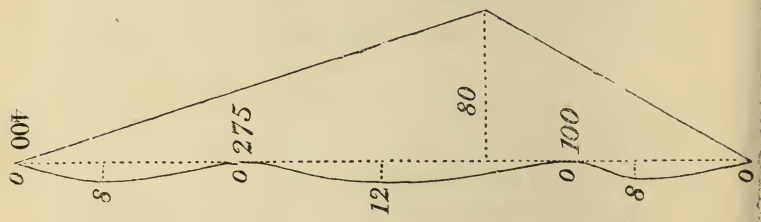
$$\begin{aligned} 25 \times 8 &= 200 \\ (8+14) \times 95 &= 2090 \\ (14+5) \times 55 &= 1045 \\ 25 \times 5 &= 125 \end{aligned}$$

2) 75700 = the sum

37850 square

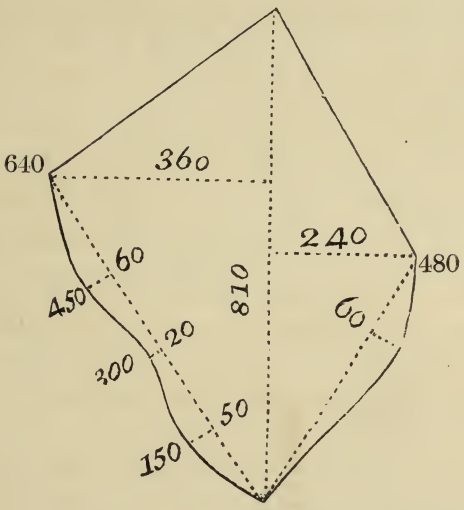
links the area; hence (37850 ÷ 100000) × 4 × 40 = 1 rood, 20·56 perches, the content of the field A B C required.

Find the content of the following field:—



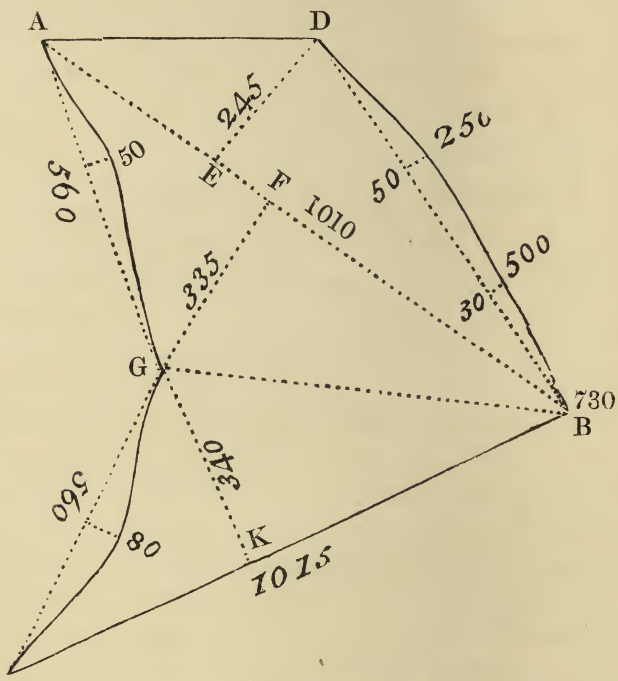
Ans. 28·72 *p.*

Find the content of the following field:—



Content, 2 *a.* 3·024 *r.*

It is frequently most convenient to go out of the bounds, and measure an offset, which must be subtracted, as in the following example :—



Content, 4 *a.* 1 *r.* 38 *p.*

By Rules to Prob. IV. VII. & VIII. *Pages* 66, 75, & 78.

Square Links.

$(DE+FG)\times AB=(245+335)\times 1010=585800$

$HB\times GK=1015\times 340=345100$

Double the area of the offsets from D to B.

$250\times 50=12500$

$(50+30)\times 250=20000$

$230\times 30=6900$

$970300=$sum

Double the area of the offsets from H to A, which deduct.

560×80

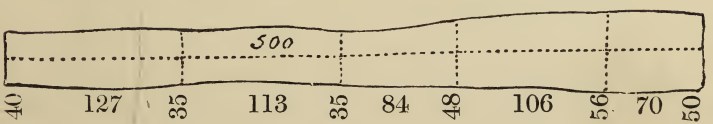
560×50 $\Big\}=72800$

$2)897500=$diff.

448750 square

links, the area; hence $(448750\div 100000)\times 4\times 40=$ 4 *a.* 1 *r.* 38 *p.* the content of the field A G H B D A.

The best method to find the content of a long narrow piece of land, of an irregular width, lying either in a straight direction or a curving one, is, by measuring a line along the middle of it, and measuring the width in different places; let these widths be treated as offsets, and multiplied into the chain-line, &c., as before directed.—*See Rule to Problem* VIII. *page* 78.



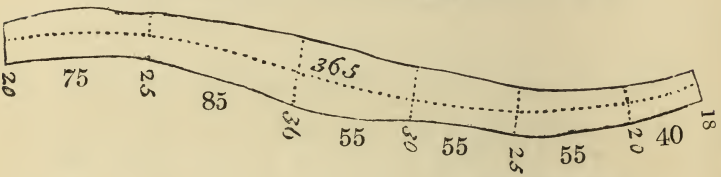
Solution. *By Rule to Prob.* VIII. *Page* 78.

$(40+35)\times 127 = 9525$
$(35+35)\times 113 = 7910$
$(35+48)\times 84 = 6972$
$(48+56)\times 106 = 11024$
$(56+50)\times 70 = 7420$

$2)\ 42851$

21425·5 square links, the area; hence $(21425{\cdot}5 \div 100000)\times 4\times 40 = 34{\cdot}2808$ *p.* the content of the irregular figure.

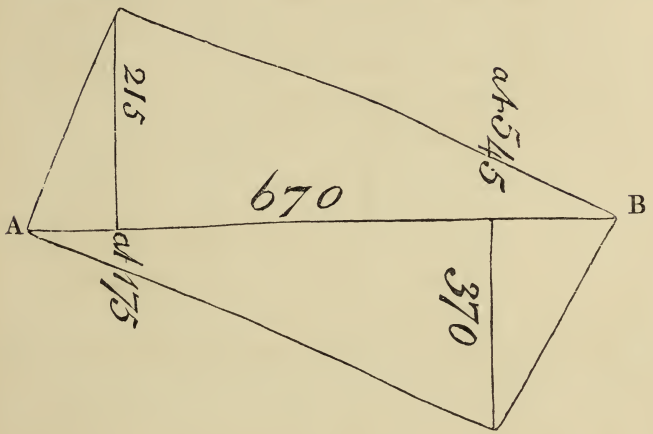
Find the content of the following piece of land:—



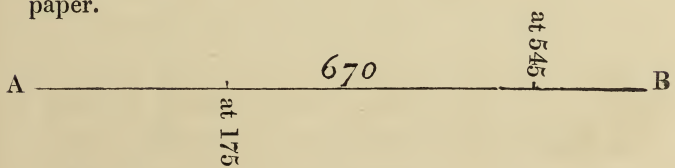
Ans. 15·368 *p.*

The foregoing rules are designed to find the content of any piece of land without drawing the plot; but the plot may be drawn from the same memorandums, by noting in what part of the base line the perpendiculars are raised: thus—

Suppose the following rough sketch to be taken in the field :—



With a pair of compasses, take the length of the diagonal A B 670 off a plotting scale, and draw it on paper.

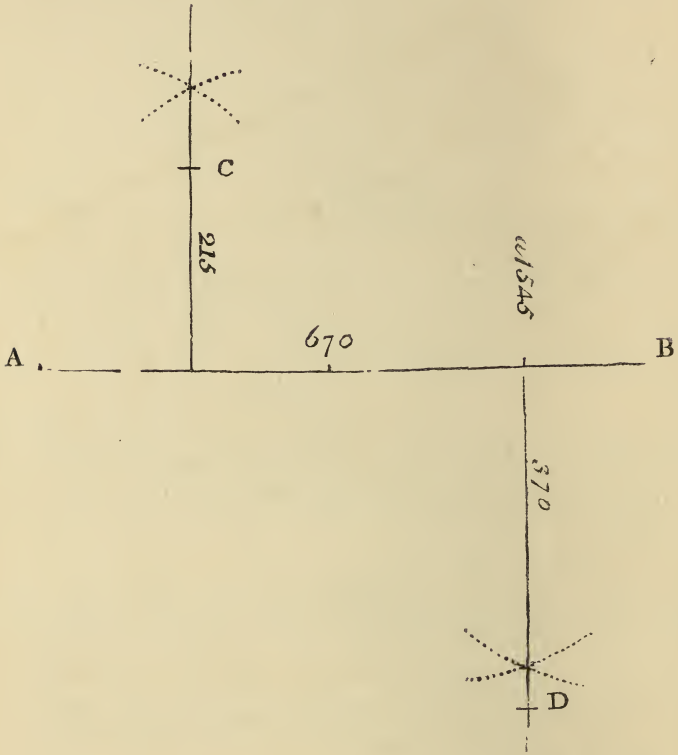


Take the distance of the first perpendicular 175, and set off from A towards B.

Take the distance of the second, and set off in the same manner.

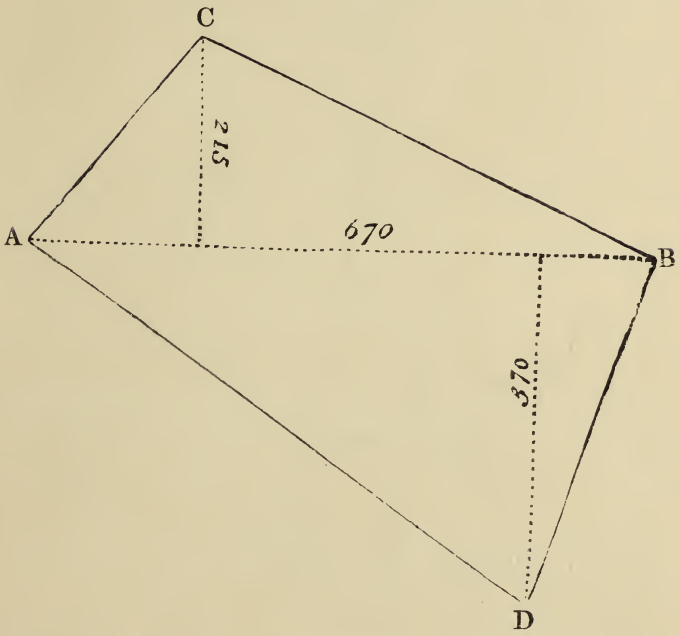
From each of these points draw a perpendicular by *Problem* II., *page* 14, or with a small square, made of brass or wood.

Make the first perpendicular 215, and the second 370.



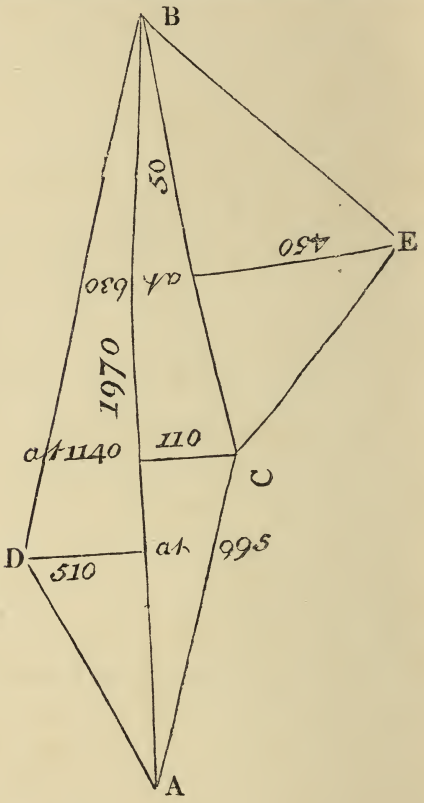
Draw the lines A C and C B.
And the lines A D and D B.

The plot will appear thus :

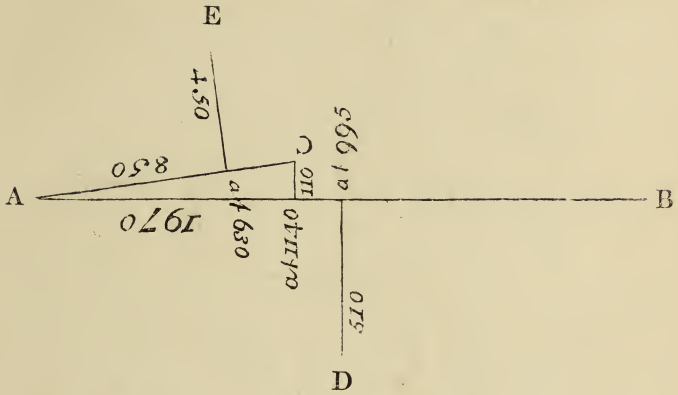


Content, 1 *a.* 3 *r.* 33·56 *p.*

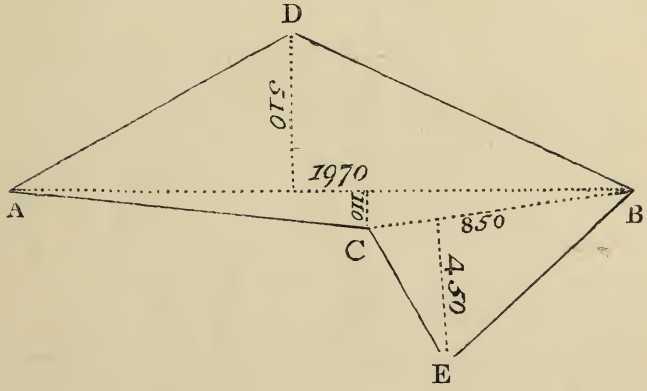
It is required to draw the plan and find the content of a field from the following rough sketch:—



FIRST OPERATION.

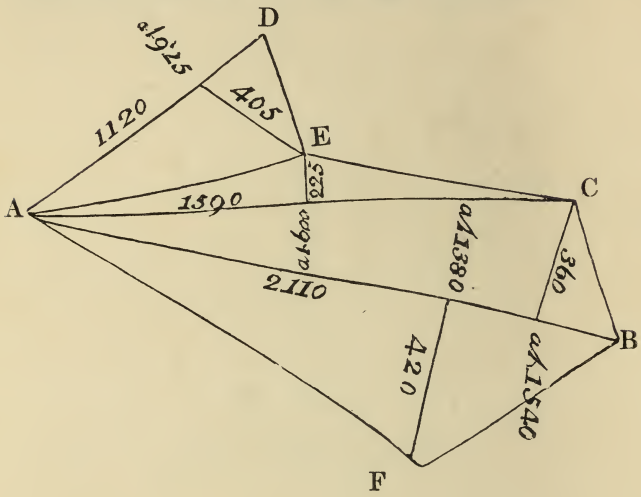


SECOND OPERATION.

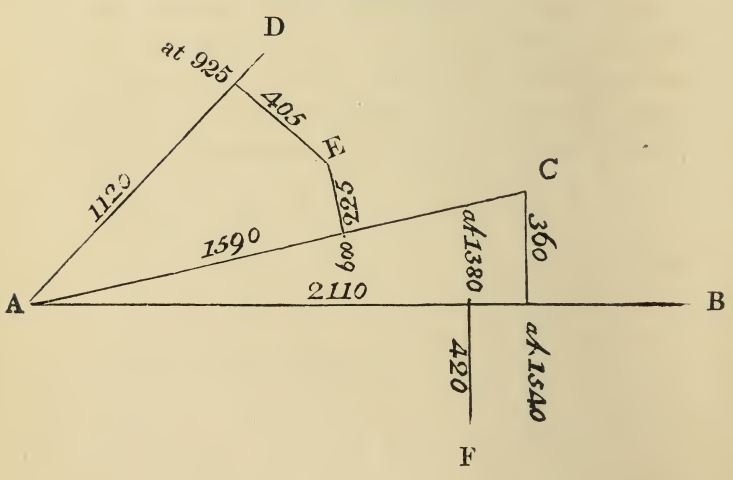


Content, 8 *a.* 0 *r.* 3·02 *p.*

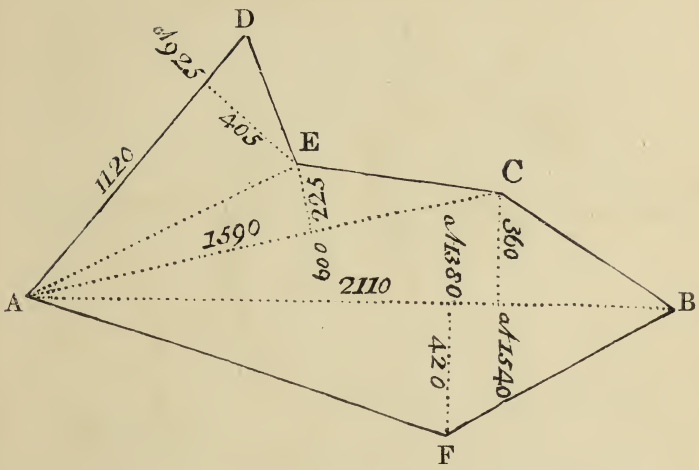
It is required to draw a plan and find the content of a field, from the following rough sketch:—



FIRST OPERATION.



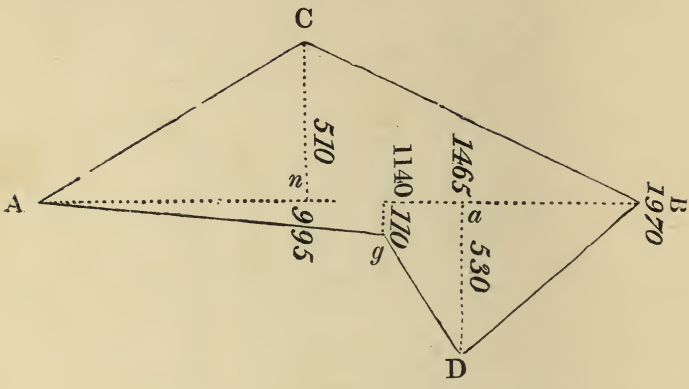
SECOND OPERATION.



Ans. 12 *a.* 1 *r.* 5·72 *p.*

Many fields may be surveyed by measuring one diagonal to each, and perpendiculars from it to the angles on each side, which will divide it into right-angled triangles or trapezoids, the content of which can be found by the offset-rule. *See page* 116.

Examples of the last, two fields, surveyed by this method.



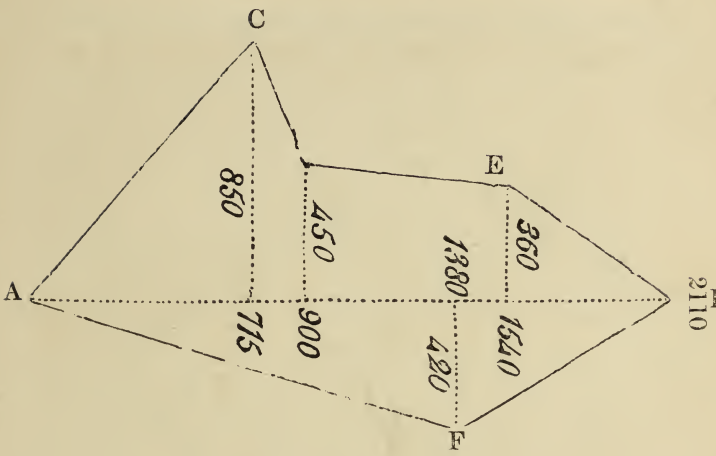
Double the Area in Square Links.

By Rule, Prob. IV. A B×C n=1970×510 =1004700
By Rule, Prob. IV. A e×g e=1140×110 = 125400
By Rule, Prob. VIII. {(D a+g e)×e a = / (530+110)× 325} = 208000
By Rule, Prob. IV. B a×D a= 505×530 = 267650

The sum=1605750 double

the area of the whole figure in square links, whose half= 802875 square links the area.

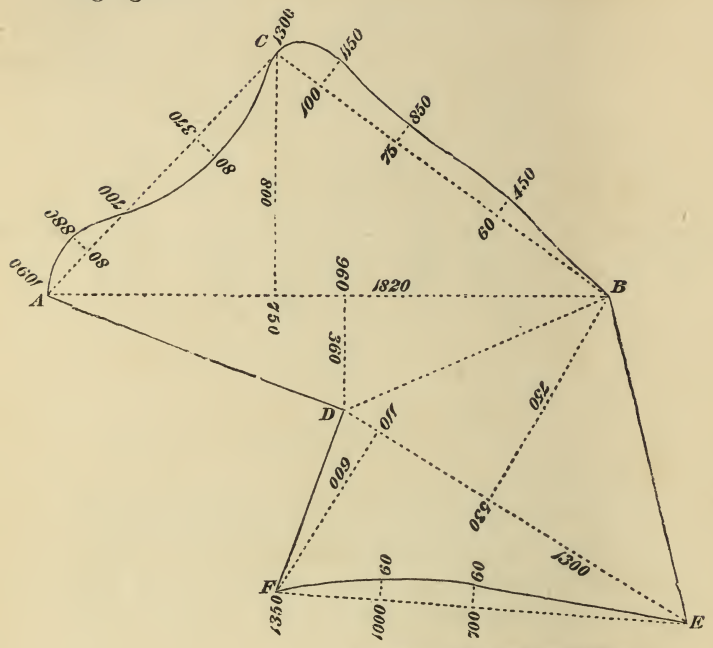
Then (802875÷100000)×4×40=8 *a.* 0 *r.* 4·6 *p.* the content of the field A C B D *g* A.



Content, 12 *a.* 1 *r.* 6·44 *p.*

If the field has offsets, the lines to which they belong must be measured as before directed; and, in the rough sketch, note down the distance of each offset from that end of the line you began to measure from.

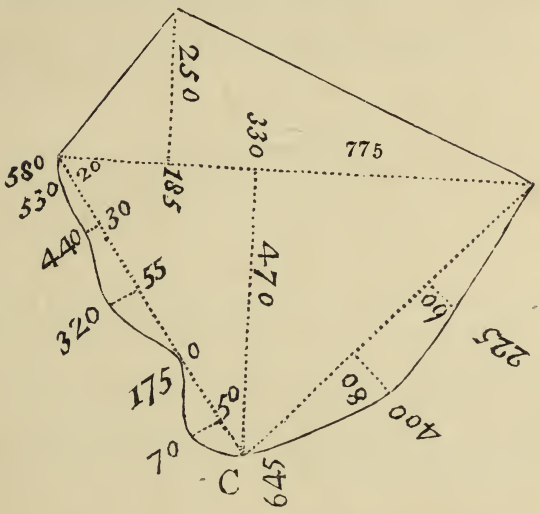
Required a correct plan and content of the following figure :—



~~Content, 20 a. 1 r. 34 1/2 p.~~

Content 19 a. 1 r. 32. 7 poles

Plan the following figure upon a larger scale, from the dimensions given therein :—



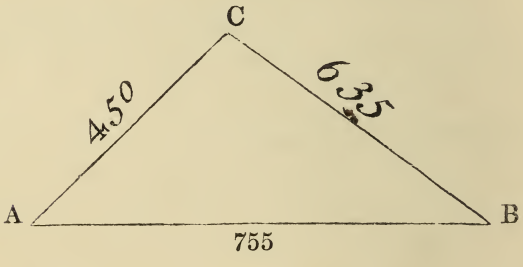
Content, 3 *a.* 0 *r.* 38·42 *p.*

PART THE FOURTH.

TO SURVEY WITH THE CHAIN.

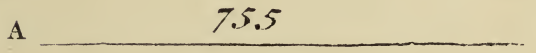
Draw a rough sketch of the field on paper, either by going round it, or by drawing the lines as they are measured.

If a triangular field is to be surveyed, measure the three sides, and note down the length of each in the rough sketch: thus—

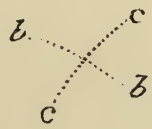


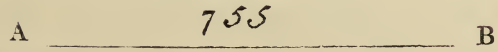
This must be accurately plotted before the content be found. To do which, take the length of the longest side off the scale with the compasses, and draw it on paper.

Take the length of another line A C 450 off the scale, set one point of the compasses on A, and draw the arc *b b*.

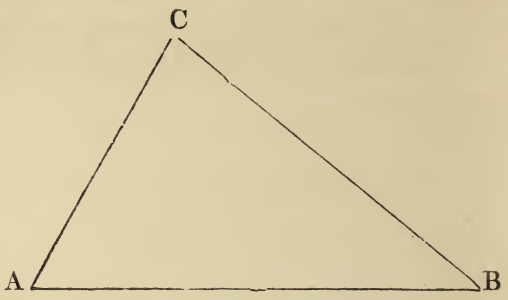


Take the length of the other line B C 635 off the scale, and, with B for a centre, draw the arc *c c*.

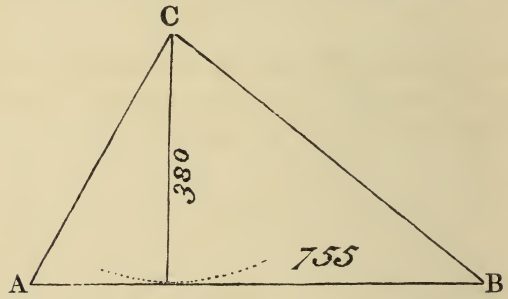




From the point of intersection draw the lines A C and B C, which will complete the plot.



Then let fall a perpendicular from the point C on the line A B, by *Problem* III. *page* 17. Measure the length of the perpendicular on the plotting scale, and it will be found to be 380 links: or, open the compasses to such a width as will describe an arc touching the base A B without passing through it.

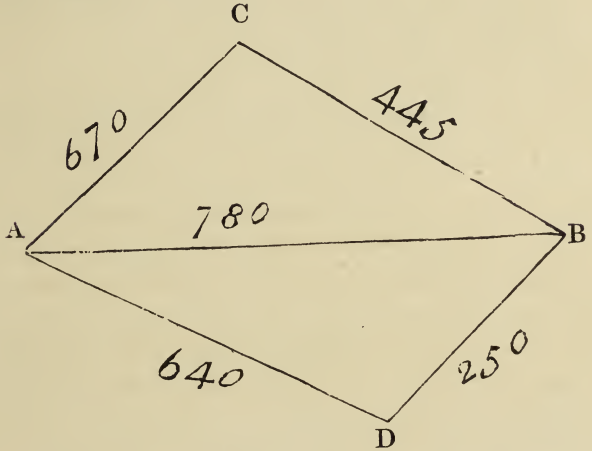


Measure that distance on the plotting scale, and it will be found the same as before, 380 links.

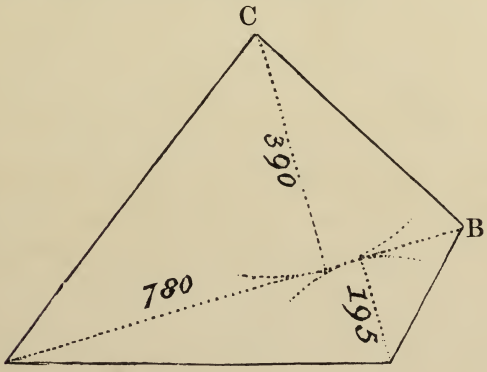
Content, 1 *a.* 1 *r.* 29·52 *p.*

To survey a Field which has four Sides.

Draw a rough sketch of the field, and a diagonal across it. Note down the length of each side, and the diagonal, thus—



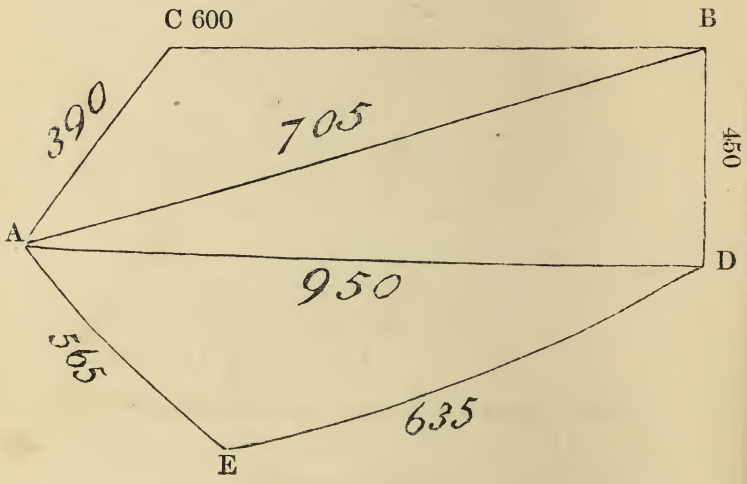
On the diagonal A B draw the two triangles A B C and A B D, from the scale, and measure a perpendicular to each, as in the last example.



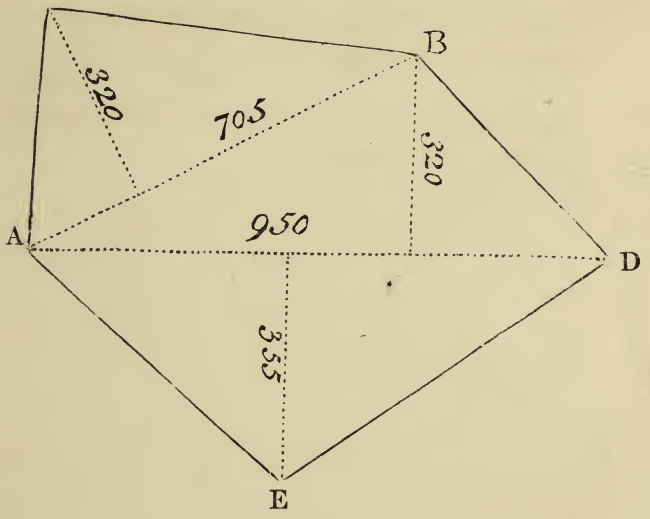
Content, 2 *a.* 1 *r.* 5·04 *p.*

Any other field may be surveyed in the same manner, by measuring as many diagonals as will divide it into triangles.

Required the plan and content of a field, from the following rough sketch:—



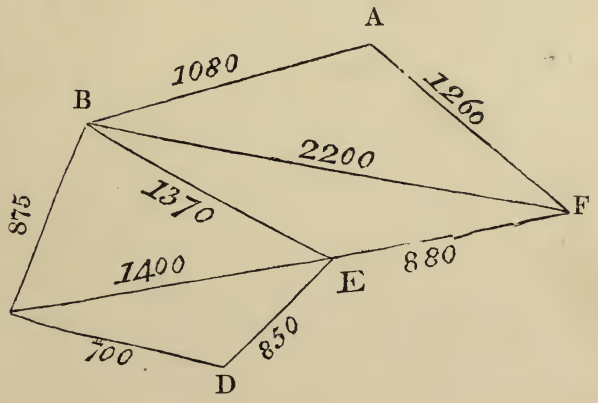
PLAN.



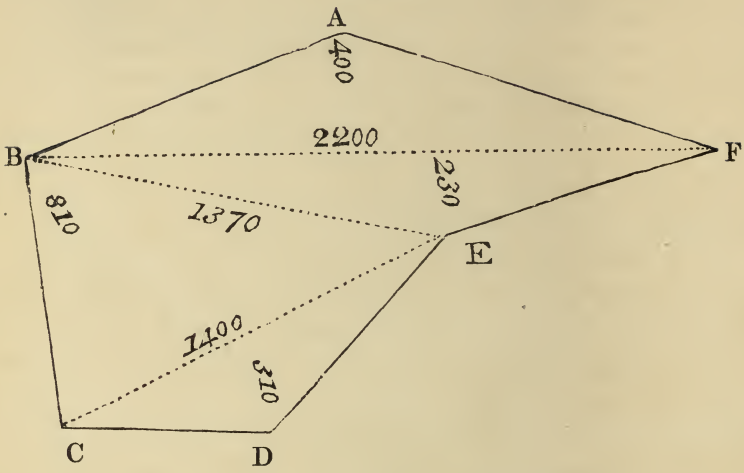
Content, 4 *a.* 1 *r.* 13·48 *p.*

Draw a plan and find the content of the following field:—

ROUGH SKETCH.

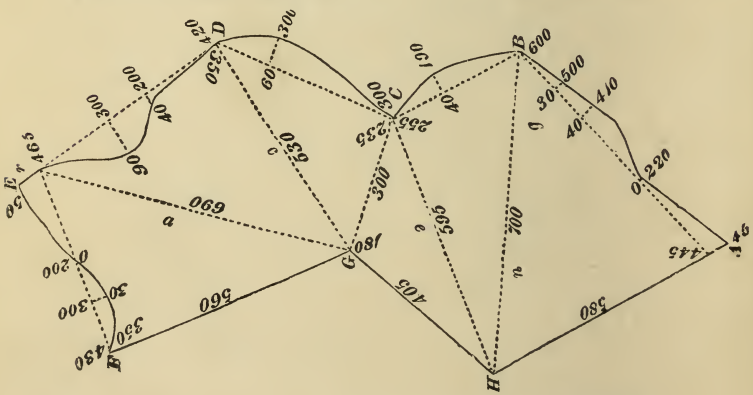


PLAN.



Content, 14 *a.* 3 *r.* 3·2 *p.*

Required a plan and content of a field from the following dimensions :—



Content, 6 *a.* 0 *r.* 28·72 *p.*

Double the measure in Square Links.

By Rule, Prob. VII. $(Fa+Da)\times Gr=(350+350)\times 690=483000$
By Rule, Prob. IV. $Cc\times G\,D=235\times 530\ldots\ldots=124550$
By Rule, Prob IV. $H\,C\times Ge=595\times 180\ldots\ldots=107100$
By Rule, Prob. VII. $(Cg+p\,n)\times H\,B\,(255+445)\times 700=490000$
By Rules, Prob. IV. *and* VII. The off-sets along A B = 26800
By Rule, Prob. IV. The off-set along B C..... = 12000
By Rule, Prob. IV. The off-set along C D..... = 25200
By Rule, Prob. IV. The off-set E*r*, along *o r*..... = 10000

The sum = 1278650

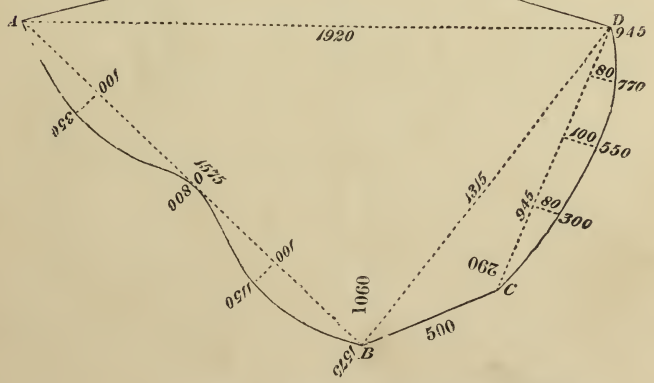
By Rules, Prob. IV. *and* VII. Double the area of the off-sets along D*r*, and *o* F, which deduct 42750

The difference = 1235900

square links, double the area of the whole field, whose half= 617950 square links the area.

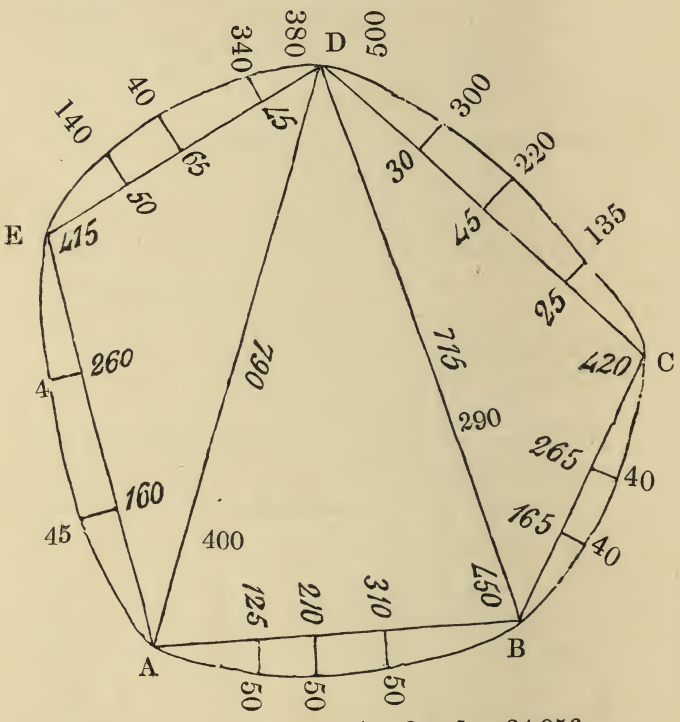
Then $(617950\div 100000)\times 4\times 40=6$ *a.* 0 *r.* 28·72 *p.*=*Answer.*

Required a plan and content of a field from the following dimensions:—



Content, 18 *a.* 2 *r.* 26·76 *p.*

Let a correct plot be drawn, and the area of a field be found, from the following figure :—



Ans. 3 *a.* 1 *r.* 34·956 *p.*

PART THE FIFTH.

LAYING OUT AND DIVIDING LAND.

PROBLEM I.

To reduce any number of Acres, Roods, and Perches into Square Links.

RULE.—Reduce the given quantity into perches, and multiply by 625 the number of square links in one perch; the product will be the square links required. Or take out the equivalents from the following Table, and the sum will be the square links required.

A TABLE FOR REDUCING ACRES, ROODS, AND PERCHES INTO SQUARE LINKS.

Acres.	Square Links.
1	100000
2	200000
3	300000
4	400000
5	500000
6	600000
7	700000
8	800000
9	900000
10	1000000
20	2000000
30	3000000
40	4000000
50	5000000
60	6000000
70	7000000
80	8000000
90	9000000
100	10000000

Roods.	Square Links.
1	25000
2	50000
3	75000

Perches.	Sq. Links.
1	625
2	1250
3	1875
4	2500
5	3125
6	3750
7	4375
8	5000
9	5625
10	6250
11	6875
12	7500
13	8125
14	8750
15	9375
16	10000
17	10625
18	11250
19	11875
20	12500

Perches.	Square Links.
21	13125
22	13750
23	14375
24	15000
25	15625
26	16250
27	16875
28	17500
29	18125
30	18750
31	19375
32	20000
33	20625
34	21250
35	21875
36	22500
37	23125
38	23750
39	24375

Example 1. Reduce 9 *a.* 3 *r.* 39 *p.* into square links.

a.	*r.*	*p.*		
9	3	39		
4				
39				
40				
1599				
625			*By the Table.*	
7995			Sq. Links.	
3198			9 *a.*=900000	
9594			3 *r.*= 75000	
999375	*Ans.*		39 *p.*= 24375	
			999375	=*Ans. as before.*

Example 2. Find the number of square links in 96 *a.* 2 *r.* 35 *p.* *Ans.* 9671875.

PROBLEM II.

To lay out, in a Square, a given quantity of Land.

Rule.—Extract the square root of the given area, and it will be the side of the square required. (*See note* 1, *page* 61.)

Example 1. Lay out, in a square, 9 *a.* 1 *r.* 28 *p.*

Square Links.

By the Table
- 9 *a.*=900000
- 1 *r.*= 25000
- 28 *p.*= 17500

Then $\sqrt{942500}$=970·8 links, the side of the square.

Make A B=970·8 links, which will represent one of the sides of the square. At A and B erect the two perpendiculars, A D and B C, each equal to A B. Measure the line C D, and if it be found=970·8 links, the work is right.

Example 2. Required the side of a square that shall contain 13 *a.* 1 *r.* 32 *p.* *Ans.* 1159·7 links.

Example 3. Lay out, in a square, 7 *a.* 1 *r.* 24 *p.* *Ans.* 860·2 links.

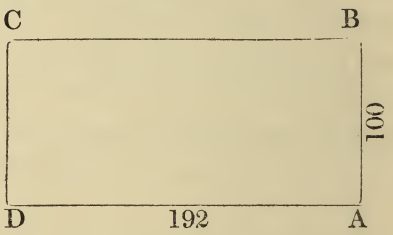
PROBLEM III.

To lay out a given quantity of Land in a Rectangular form, having one side given.

RULE.—Divide the proposed area by the length of the given side, and the quotient will be the length of the required side. (*See Note* 1, *page* 62.)

Example 1. It is required to lay out 120 acres in a rectangular form, the length of one side being given, equal 100 perches.

First, 120 *a.* = 19200 *p.*, and 19200 ÷ 100 = 192 *p.*, the length of the side sought.



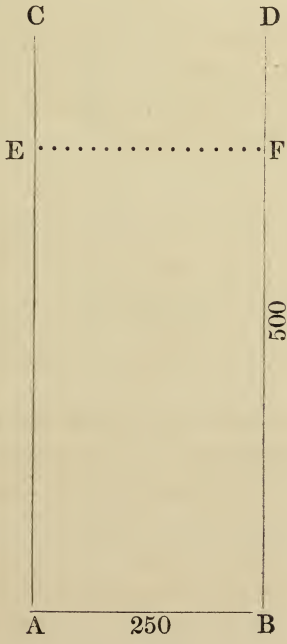
Make A B = 100 *p.* = the given side; at A and B erect the perpendiculars A D and B C each = 192 *p.* Then measure C D, and if it be found = A B, the work is right.

Example 2. What length of a rectangular field, whose breadth is 750 links, will make 2 *a.* 3 *r.* 25 *p.*?

Ans. 387½ links.

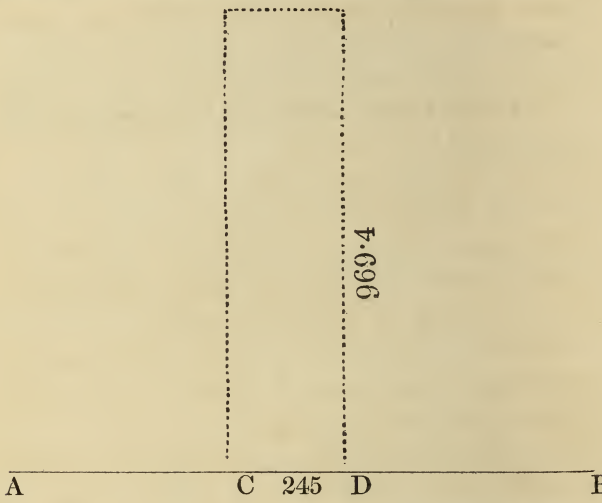
Note. If it be required to cut off a certain portion from a rectangle by a line parallel to one side, it will be merely necessary to divide the square links in the quantity required to be cut off by the links in the given side, and the quotient will show the distance on the other sides where the line of division should be drawn.

Example 1. If the side A B of a rectangle is 250 links, what length must be taken on the lines A C and B D, to part off an acre and a quarter from the rectangle?



First, $1\frac{1}{4}$ *a.*$=125000$ sq. links, and $125000 \div 250 =$ 500 links$=$A E or B F.

Example 2. It is required to part off 2 *a.* 1 *r.* 20 *p.* on part of the line A B, *viz.* from C to D, 245 links long, in the form of a right-angled parallelogram.



Ans. 969·4 links.

Example 3. Part off 1 *a.* 3 *r.* 24 *p.* on part of the line A B, *viz.* from C to D, 250 links long, in the form of a rectangle. *Ans.* 760 links.

PROBLEM IV.

To lay out a given quantity of Land in a Rectangular form, having the length to the breadth in a given ratio.

RULE.—As the less number of the given ratio
Is to the greater,
So is the given area
To a fourth term.

The square root of this fourth term will be the length required. Having the length, the breadth may be found by the preceding problem.

Example 1. It is required to lay out 864 acres in a rectangular form, having the length to the breadth in the ratio of 5 to 3.

First, 864 *a.* = 138240 *p.*; also as 3 : 5 : : 138240 : 230400, and $\sqrt{230400} = 480$ *p.*, the length required; hence $138240 \div 480 = 288$ *p.*, the breadth required.

Example 2. It is required to lay out 27 *a.* 3 *r.* 20 *p.* in a rectangular form, having the length to the breadth in the ratio of 9 to 7.

Ans. Length = 75·725 *p.*, and breadth = 58·897 *p.*

PROBLEM V.

To lay out a given quantity of Land in a Rectangular form, having the length to exceed the breadth by a given difference.

RULE.—To the given area, add the square of half the given difference of the sides, and extract the square root of the sum; to this root add half the given difference for the greater side, and subtract it therefrom for the less.

Example 1. It is required to lay out 47 *a.* 2 *r.* 16 *p.* in a rectangle, of which the length is to exeeed the breadth by 80 perches.

First, 47 *a.* 2 *r.* 16 *p.* = 7616 *p.*, and $80 \div 2 = 40$; hence $\sqrt{(7616 + 40^2)} = \sqrt{(7616 + 1600)} = \sqrt{9216} = 96$. Also, $96 + 40 = 136$ *p.* = length, and $96 - 40 = 56$ *p.* = breadth.

Example 2. It is required to lay out 114 *a.* 2 *r.* 33·4 *p.* in a rectangular form, having the length to exceed the breadth by 15·10 chains.

Ans. Length = 42·25 ch.; breadth = 27·15 ch.

Example 3. Lay out 7 *a.* 0 *r.* 24 *p.* in a rectangle, so that the length may exceed the breadth by 18 perches.

Ans. Length = 44 *p.*; breadth = 26 *p.*

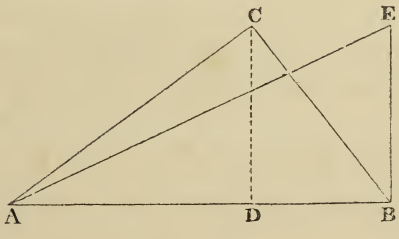
PROBLEM VI.

Upon a given Base, to lay out a Triangle that shall contain any given number of Acres, &c.

Rule.—Divide twice the area by the base, and the quotient will be the perpendicular of the triangle.

Example 1. Lay out 3 *a.* 2 *r.* 16 *p.* in the form of a triangle, the base of which must be 1200 links.

First, 3 *a.* 2 *r.* 16 *p.* = 360000 sq. links; and, by the rule, $360000 \times 2 \div 1200 = 720000 \div 1200 = 600$ links, the perpendicular.



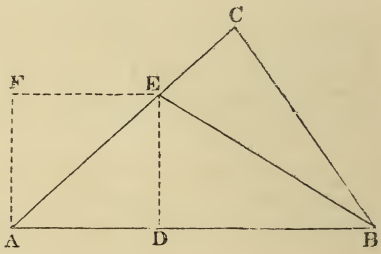
Upon the given base A B, as at D, erect the perpendicular D C, which make = 600 links; then stake out the lines A C and B C; A B C will be the required triangle. If the perpendicular be erected at either end of the base, as at B, the line A E

must be staked out, and A B E will be the triangle sought.

Example 2. Required the perpendicular of a triangle, which contains 6 *a.* 2 *r.* 37 *p.*, its base being 1556 links. *Ans.* 865·2 links.

Note. The same rule is applicable, if it be required to part from a triangle, upon the base or longest side, any proposed quantity of land, by a line drawn from either of the angles at the base, to the opposite side.

Example 1. Let A B C represent a triangle, whose base A B is 1200, and sides A C and B C, 1000 and 800 links respectively; it is required to part off 2 *a.* 2 *r.* 24 *p.* by a line drawn from the angle B to the side A C.



First, 2 *a.* 2 *r.* 24 *p.* = 265000 square links; and (265000 × 2) ÷ 1200 = 530000 ÷ 1200 = 441·7 links, the perpendicular D E.

At A erect the perpendicular A F, which make = 441·7 links; draw F E parallel to A B, intersecting

the side A C in the point to which the division fence B E must be made.

Example 2. From a triangular field, whose sides are 1500, 1200, and 1000 links respectively, part off 3 *a.* 2 *r.* 16 *p.* by a fence made from the greater angle at the base to the opposite side.

Ans. The perpendicular is = 480 links.

PROBLEM VII.

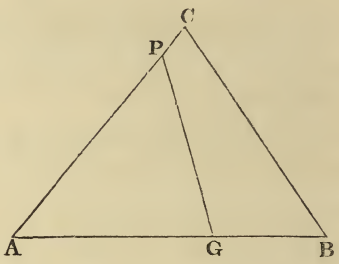
The Area and two Sides of a Triangle being given, to cut off a Triangle containing a given Area, by a Line running from a given Point in one of the given Sides and falling on the other.

RULE.—As the given area of the triangle
Is to the area of the part to be cut off,
So is the rectangle of the given sides
To a fourth term.

Divide this fourth term by the distance of the given point from the angular point of the two given sides; the quotient will be the distance of the required point from the same angle.

Example 1. Given the area of the triangle A B C 5 acres, the side A B 50 perches, the side A C 40 perches, and the distance of a point P from the angle A, 36 perches; it is required to find a point G

to which, if a line be drawn from the point P, it shall cut off a triangle A P G, containing 3 *a.* 0*r.* 20*p.*



The area of the triangle A B C=5 *a.*=800 square perches, the area of the triangle A P G=3*a.* 0*r.* 20*p.*= 500 square perches, and A B × A C=50 × 40=2000; hence, *by the rule,* 800 : 500 : : 2000 : 1250, the fourth term, which, divided by 36=A P, gives 34·72=A G.

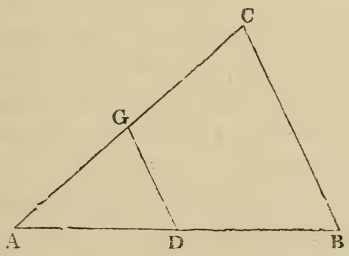
Example 2. Given the area of the triangle A B C, 12 *a.* 1*r.* 23*p.*, the side A B 20 chains, the side A C 16·25 chains, and the distance of a point P in the side A C, from the angle A 8·5 chains; it is required to find the distance A G of a point G in the line A B, so that a line drawn from P to G may cut off a triangle A P G, containing 3 acres. *Ans.* 9·26 chains.

PROBLEM VIII.

The Area and Base of a Triangle being given, to cut off a Triangle containing a given Area by a Line running parallel to one of the Sides.

RULE.—As the given area of the triangle
Is to the area of the triangle to be cut off,
So is the square of the given base
To the square of the required base,
The square root of which will be the base of the required triangle.

Example 1. Given the area of the triangle A B C 500 square perches, and the base 40 perches; it is required to cut off 120 square perches towards the angle A, by a line D G running parallel to the side B C.



As 500 : 120 : : A $B^2 = 40^2 = 1600 : 384 =$ A D^2 ; hence, A D $= \sqrt{384} = 19{\cdot}6$ perches.

Make A D $= 19{\cdot}6$ perches, then draw D G parallel to B C, and the work will be completed.

Example 2. Given the area of the triangle A B C 10 acres, and the base A B 25 chains, to find A D a part of the base, so that a line D G running from the point D, parallel to the side B C, may cut off a triangle A D G, containing $4\frac{1}{2}$ acres.

Ans. A D $= 16{\cdot}77$ chains.

PROBLEM IX.

To lay out a Trapezium that shall contain any number of Acres, &c.; having one of its Sides as a Base Line given.

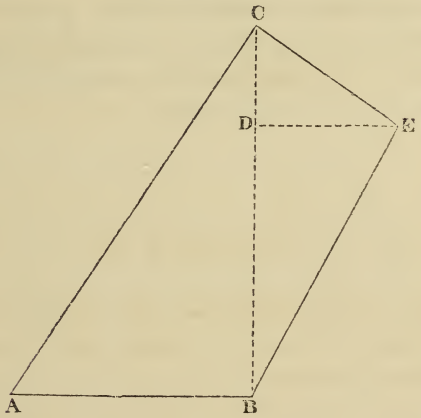
Rule.—Divide the given area into two parts, either equal or unequal; and then, by Problem VI., find the perpendicular that will lay out one of these parts in a right-angled triangle, upon the given base. Take this perpendicular as one of the diagonals of the trapezium, and also as the base upon which to lay out the other triangle.

Example 1. Lay out 8 acres in a trapezium, upon a given side of 800 links.

Divide the area into 5 and 3 acres, and let the triangle upon the given side contain the 5 acres. Now

5 *a.*=500000 square links, and 500000 × 2 ÷ 800 = 1250 links, the perpendicular of the first triangle, and also the base of the second.

Again: 3 *a.*=300000 square links, and 300000 × 2 ÷ 1250 = 480 links, the perpendicular of the second triangle.



Let A B = 800 links, represent the given side. At B erect the perpendicular B C = 1250 links. Then from any point D in the line B C erect the perpendicular D E = 480 links. The four outlines being properly staked out completes the trapezium.

Example 2. Lay out 12 acres in a trapezium, upon a side of 1400 links.

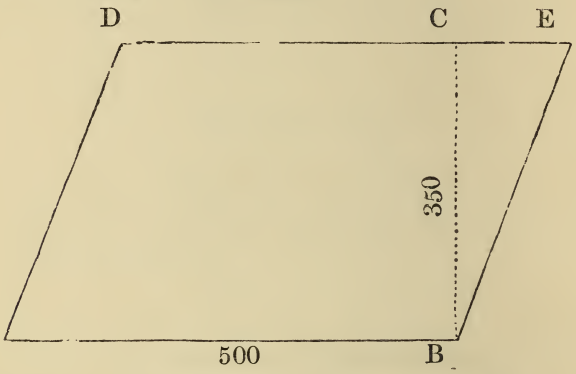
Ans. Dividing the area into 7 and 5 acres, the perpendicular of each triangle is found to be 1000 links.

PROBLEM X.

To lay out a given quantity of Land in the form of a Parallelogram, the Fences not being at Right Angles.

RULE.—Divide the area by the base, and the quotient will be the perpendicular.

Example 1. Lay out 1 *a.* 3 *r.* on the base A B = 500 links, of a parallelogram, the fences A B and A D not being at right angles.



First 1 *a.* 3 *r.* = 175000 sq. links, and 175000 ÷ 500 = 350 links, the perpendicular.

Draw B C perpendicular to A B, and equal to 350 links. Measure the perpendicular C D to the line B C. Produce D C to E till D E = A B, and join B E. The parallelogram A B E D will contain the given quantity of land.

Example 2. Lay out 1 *a.* 2 *r.* 28 *p.* in the form of an oblique parallelogram, the base being equal to 300 links. *Ans.* Perp. = $558\frac{1}{3}$ links.

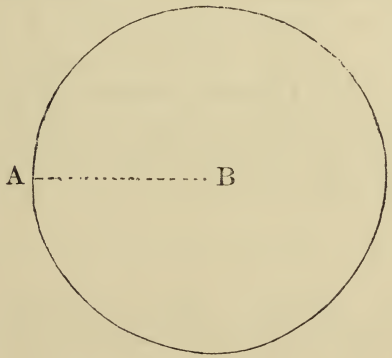
PROBLEM XI.

To lay out a given quantity of Land in a Circle.

RULE.—Divide the area by ·7854, and extract the square root of the quotient for the diameter.

Example 1. Lay out an acre of land in a circle.

First 1 acre = 100000 square links, and $\sqrt{(100000 \div \cdot 7854)} = \sqrt{127323 \cdot 65} = 356 \cdot 82$ links, the diameter required.



In laying out the circle, take a strong cord, in length equal to the radius of the circle, which, in this case, will be 178·4 links. Fix one of the ends at B, as a centre, and attach the other to the lower extremity of the off-sett staff. Stretch the cord to A, and describe the circle, keeping the staff perpendicular.

Example 2. Required the diameter of a circle that will contain half an acre of land.

Ans. 252·3 links.

PROBLEM XII.

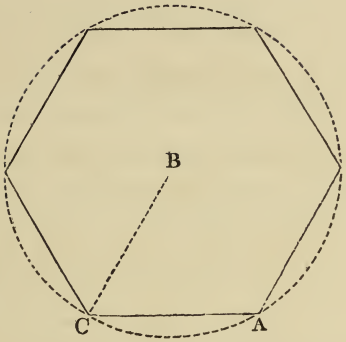
To lay out a given quantity of Land in a regular Polygon.

RULE.—Multiply the square root of the given area by that number in TABLE X., *page* 93, which is opposite the number of sides, and in the column of **SIDES**; the product will be the length of the side required. Also, to find the radius of the circumscribing circle, see *Problem* X., *Rule* V., *page* 86.

Note.—The tabular numbers need only to be used to the nearest fourth decimal place.

Example 1. Lay out one acre of land in a regular hexagon.

Here $\surd(100000) \times \cdot6204 = 316\cdot228 \times \cdot6204 = 196\cdot188$ links, the side of the required polygon; also, when the side is given, this tabular multiplier for the radius of the circumscribing circle is 1, *(see Rule* IV., *page* 86,*)* the radius of the circle circumscribing the required polygon is $=196\cdot188$ links; and, therefore, equal to the side of the polygon.—*(Euclid,* 15 *Prop.* 4 *Book, Corol.)*



To lay out the hexagon, draw the circumscribing circle as directed in the last Problem. Apply the radius B C, which is equal to the side of the hexagon, six times round the circumference, and form the polygon.

Example 2. Lay out half an acre of land in a regular octagon.

Answer. The side of the required octagon is 101·76, and the radius of its circumscribing circle 132·898 links.

Example 3. Lay out 1 *a.* 2 *r.* of land in a regular pentagon.

Ans. The side of the pentagon is 295·312, and the radius of its circumscribiug circle 251·3 links.

PROBLEM XIII.

To lay out a given quantity of Land in an Ellipse, having one of the Diameters given.

RULE.—Divide the area by ·7854, and the quotient thus arising by the given diameter, and this latter quotient will be the diameter required.

Some general properties of the ellipse.

1. The sum of the two lines drawn from the foci F, *f*, to any point in the circumference, is equal to the transverse diameter; that is, F m + *f* m = A B.

2. The square of the distance of the focus from the centre is equal to the difference of the squares of the semi-diameters; that is, O F^2 = A O^2 − C O^2, or O F = $\sqrt{(A\ O^2 - C\ O^2)}$.

3. The distance between the two foci is a mean proportional between the sum and difference of the transverse and conjugate axis.

VIZ. A B + C D : F *f* :: F *f* : A B − C D.

4. The rectangle of the distance of the focus from each vertex is equal to the square of the semi-conjugate.

VIZ. A F × F B = C O^2.

5. The transverse axis is to its conjugate, as the area of a circle, whose diameter is the transverse, is to the area of the ellipse.

6. Every parallelogram circumscribing an ellipse, is equal to a rectangle formed by the transverse and conjugate axes.

7. The area of an ellipse is to the rectangle of its two axes, as the area of any circle is to the square of its diameter.

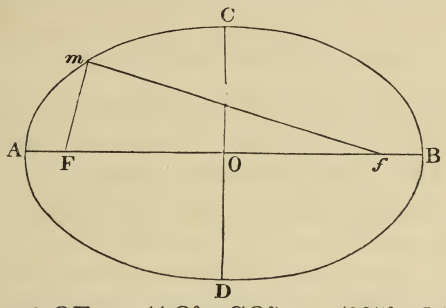
8. The transverse axis is to the conjugate, as the area of a circle, whose diameter is the transverse, is to the area of the ellipse.

9. The areas of ellipses are to each other, as the rectangles of their transverse and conjugate axes.

10. The square root of the product of the transverse and conjugate diameters, will be the diameter of a circle equal in area to the ellipse.

Example 1. Lay out an ellipse, which shall contain one acre, with a transverse diameter of 450 links.

Here $100000 \div 7854 = 127323{\cdot}6$, and $127323{\cdot}6 \div 450 = 283$ links = C D, the conjugate diameter.



By note 3, $OF = \sqrt{(AO^2 - CO^2)} = \sqrt{(225^2 - 141{\cdot}5^2)} = \sqrt{30602{\cdot}75} = 175$ links = O *f*; and $225 - 175 = 50$ links = A F or B *f*.

Procure a cord, and upon it make two loops, so that the distance between them may be equal to the transverse diameter. Measure the diameter A B, also A F, B *f*; and put down a stake at each focus.

Put the two loops over the stakes at F, *f*, and stretch the cord so that the two parts, F *m*, *f m*, may be equally tight; at *m* put down a stake at one point in the circumference of the ellipse; and, in the same manner, determine as many others as may be thought requisite.

Example 2. Lay out an ellipse which shall contain 8 *a*. 3 *r*. 8 *p*., one of the diameters given being equal to 800 links.

Ans. The other diameter is 1401 links.

Example 3. Lay out an ellipse which shall contain 4 acres, with a conjugate diameter of 600 links.

Ans. The transverse diameter is 848·82 links.

PROBLEM XIV.

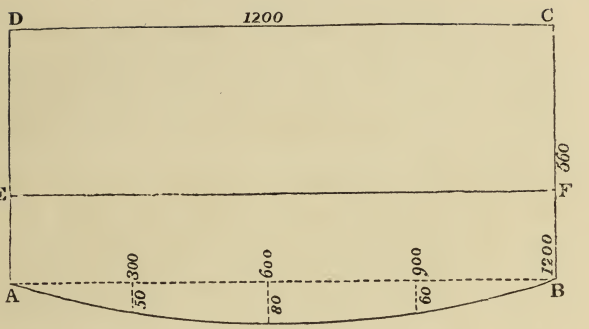
To part from a Rectangle or Triangle any proposed Quantity of Land, upon a Line, on which there are Offsets, when the Area of those Offsets is to be considered as Part of the Portion to be parted off.

RULE.—Find the area of the offsets, which subtract from the given portion, and proceed as directed in the preceding problems.

But, in a rectangle, when there are offsets on one, or both of the lines adjoining that upon which the given quantity is to be parted off, reject these offsets, and proceed as before directed.

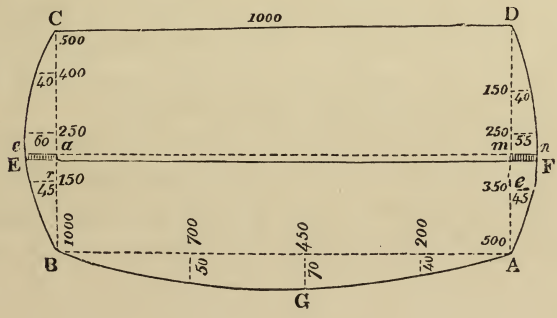
Having found the distance that the line of division must be from that upon which the given quantity is to be parted off, find the area of the offsets contained between those lines, which area divide by the latter line; and the quotient will be the distance by which the former line must be approximated to the latter.

Example 1. From a rectangular field, whose dimensions are noted in the following diagram, part off 2 *a.* 3 *r.* 32 *p.* upon the chain line, A B, so that the offsets taken upon that line may be included: A B being = 1200 links, and B C = 560.



First 2 *a.* 3 *r.* 32 *p.*=295000 square links, and the area of the offsets = 57000 square links; hence, (295000 − 57000) ÷ 1200 = 238000÷1200 = 198·4 links=A E or B F. Hence, the irregular figure A B F E contains 2 *a.* 3 *r.* 32 *p.*

Example 2. From a rectangular field, whose dimensions are noted in the following diagram, part off 2 *a.* 2 *r.* 8 *p.* by a line parallel to the chain-line A B; so that the offsets taken upon this line, and also those upon the two adjoining lines, contained between the chain-line A B and the line of division, may be included; A B being=1000 links, and A D=B C=500.



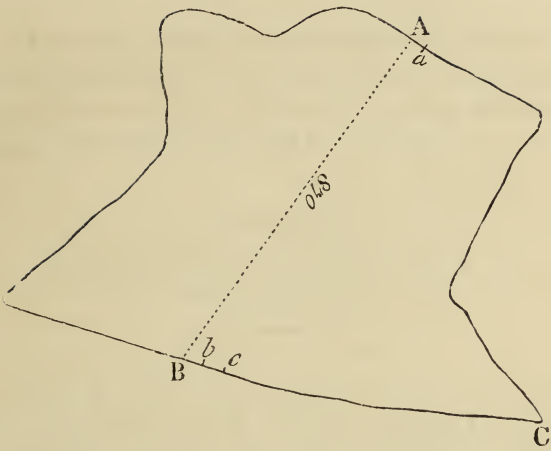
Here 2 *a.* 2 *r.* 8 *p.* = 255000 square links, and the area of the offsets taken on A B = 40250; hence, (255000 − 40250) ÷ 1000 = 214750 ÷ 1000 = 214·75 links = B *a* or A *m*, which we may call 215 links. Now 215 − 150 = 65 = *r a* = *c m*, and by the scale *a e* is found to measure 58, and *m n* 53 links; hence, the area of the offset B *a e* + the area of the offset A *m n* = 13282, which, divided by 1000, gives 13 links, the distance by which the line *e n* must be approximated to A B. Consequently, E F is the true line of division; and the irregular figure A G B E F contains 2 *a.* 2 *r.* 8 *p.* minus the two shaded offsets.

PROBLEM XV.

To divide an irregular Field into two equal Parts.

Rule.—Make a correct plan, and draw the line of division as nearly as you can in the required direction. Find the area of the whole and of one of the parts in links; and subtract the latter area from half the area of the whole. Divide the remainder by the length of the parting line, and the quotient will be the length that the line must be removed.

Example. The following field is plotted from a scale of 4 chains to an inch, and contains 6 *a.* 1 *r.* 18 *p*; it is required to divide it into two equal parts, somewhere in the direction of A B.



Divide it as nearly as possible by the line A B by inspection.

The area of the whole is 636250 links, half the area 318125 links.

The area of the part A B C is 350412 links. From which subtracted half the area, 318125, leave 32287 links.

This divided by 870, the length of A B, gives 37 links.

Remove A 37 links to *a*, and B 37 links to *b*.

If a line be drawn from *a* to *b*, the two parts will be equal.

If it is required to remove only one of the points (B for instance), let it be removed double the distance, 74 links, to *c*.

By this rule a field may be divided into any number of equal parts.

PART THE SIXTH.

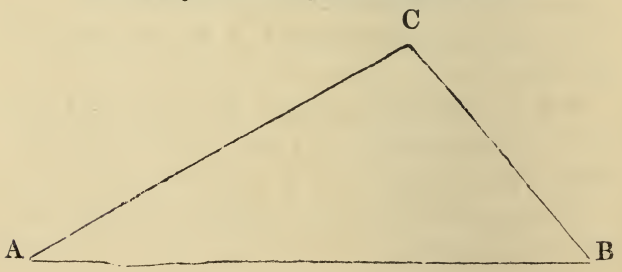
TO SURVEY SEVERAL FIELDS TOGETHER WITH THE CHAIN.

The foregoing methods of surveying are best adapted for single fields; but if several fields, or a lordship, is to be surveyed, the field-book must be kept in a different manner.

To render this method as easy to be understood as possible, I shall give examples of single fields first, and then of several together.

The pages of the field-book must be ruled into three columns: it is best to begin taking notes at the bottom of the last page, and write upwards.

To survey the Triangular Field, A B C.



Place three marks at the angles A, B, and C; begin to measure at A towards B; allow the length of A B to be 1785. The notes in the field-book will stand thus:—

Left of B
1785 A B begin at A

Measure B C, allow its length to be 907; write this above the last notes: thus—

Left of C
907 B C L of B
1785 A B begin at A

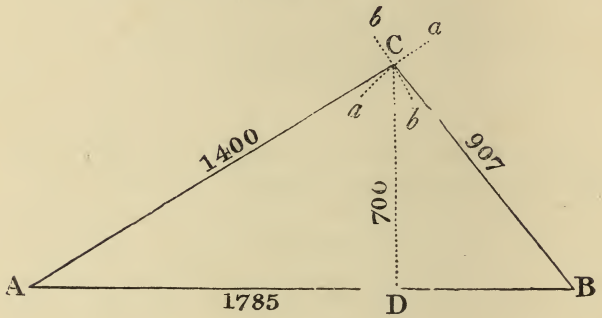
Measure the other line C A 1400, which write above the last notes: thus—

End 1400 C A L of C
907 B C L of B
1785 A B begin at A

From the preceding notes, the plan and content of the field may be obtained.

To construct the Plan.

Draw the line A B 1785.



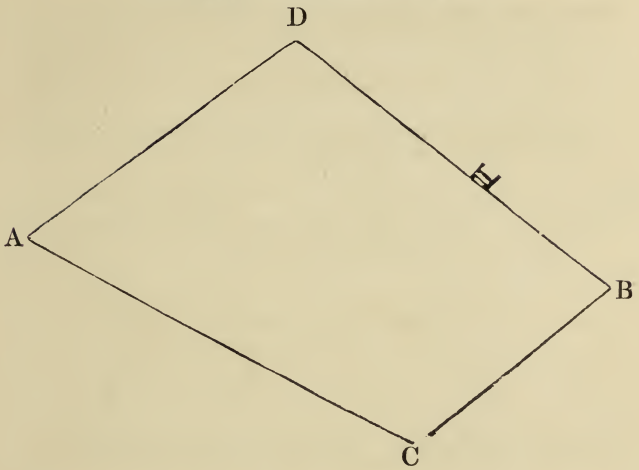
With the radius B C 907, and B as a centre, draw the arc *a a* on the left side of the line A B.

With the radius C A 1400, and A as a centre, draw the arc *b b*. A B C is the plan.

Then draw the perpendicular line C D, and it will be found by the scale to be 700 links.

The content is 6 *a.* 1 *r. nearly.*

To survey a Trapezium, A D B C.



Measure the four sides, and the diagonal A B, in the same manner as the three sides of the triangle were measured. The notes will stand thus :—

	865 D B 470 Gate R^{t} of D
	745 A D R^{t} of A
	945 C A R^{t} of C
	530 B C R^{t} of B
Diagonal Begin at	1260 A B A

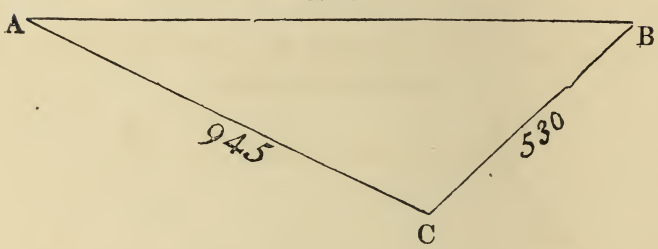
To construct the Plan.

Draw the diagonal A B 1260.

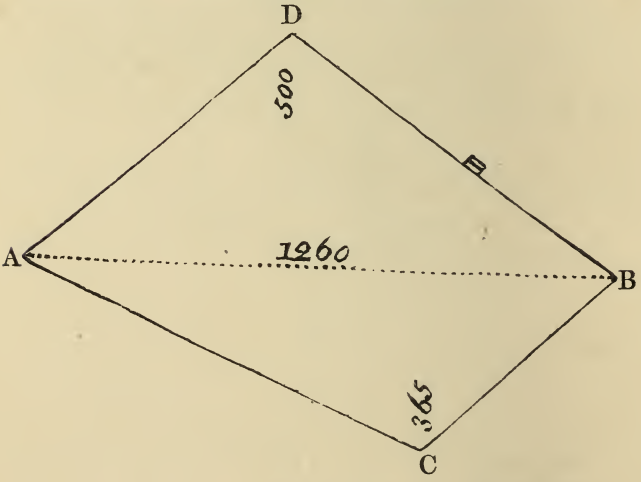
1260

On the right side of it complete the triangle A B C.

1260



On the left side complete the triangle A D B.

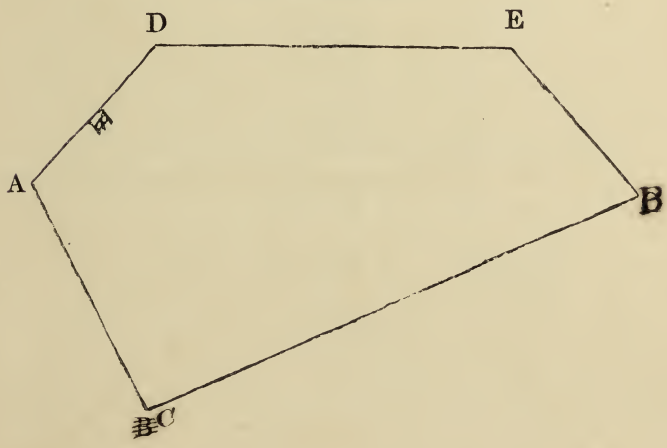


Content, 5 *a.* 1 *r.* 32 *p. nearly.*

It is required to draw the plan and find the area of a four-sided field (or trapezium) A B C D. The diagonal from A to C=18 chains, 88 links, or, which is the same thing, 18·88 chains, and the perpendiculars D E=4 chains, and B F=11 chains.

Ans. 14 *a.* 0 *r.* 25·6 *p.*

To draw a Plan and find the Content of a Field which has five Sides.

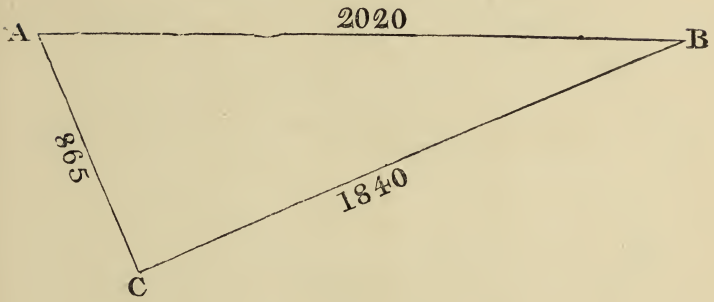


Measure the five sides and the two diagonals. The notes in the field-book will stand thus :—

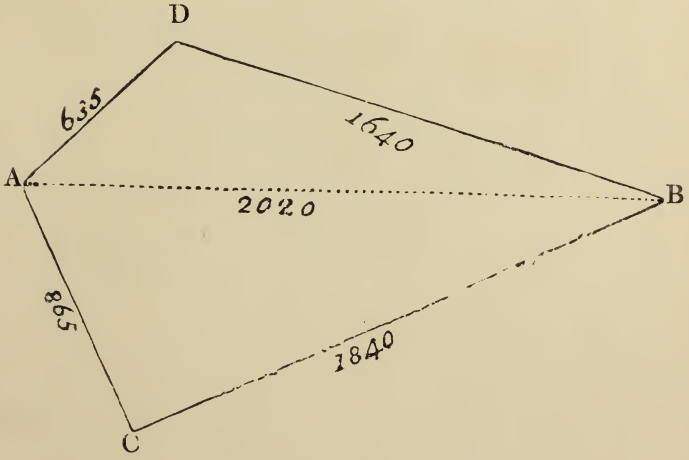
	End 1225 E D L of E
	555 B E L of B
Diagonal	1640 D B R^{t} of D
	635 A D 300 Gate R^{t} of A
	865 C A R^{t} of C
	1840 B C R^{t} of B
Diagonal Begin at	2020 A B A

To draw the Plan.

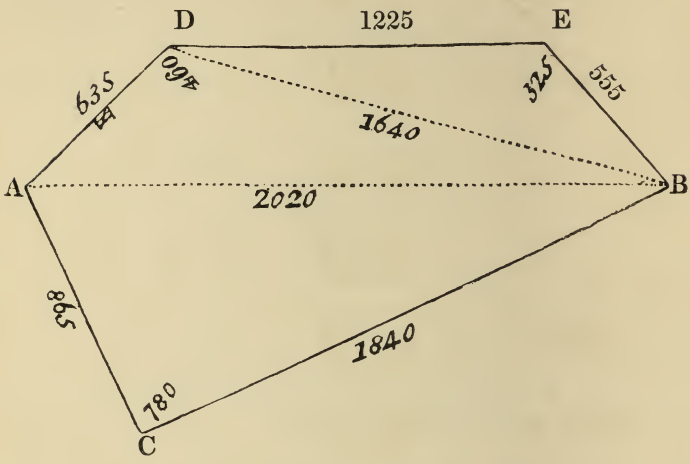
Construct a triangle with the first three lines.



With A B for a base, and the two following lines, construct another triangle, A B D.



With D B as a base, and the remaining two sides, construct the third triangle, D B E.

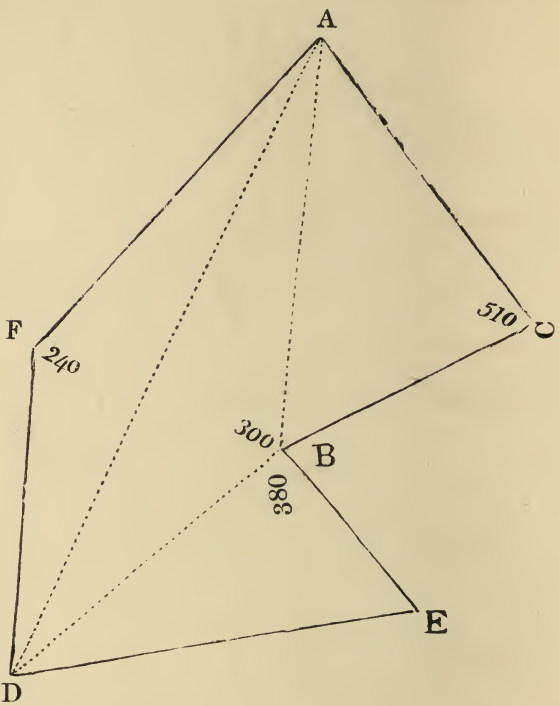


Content, 15 *a.* 0 *r.* 30·24 *p.*

Draw a plan and find the content of a field from the following notes:—

	875 F A R^{t} of F
	700 D F R^{t} of D
	860 E D R^{t} of E
	435 B E R^{t} of B
Diagonal	745 D B L of D
Diagonal	1495 A D L of A
	746 C A L of C
	600 B C L of B
Begin at Diagonal	878 A B A

Plan.

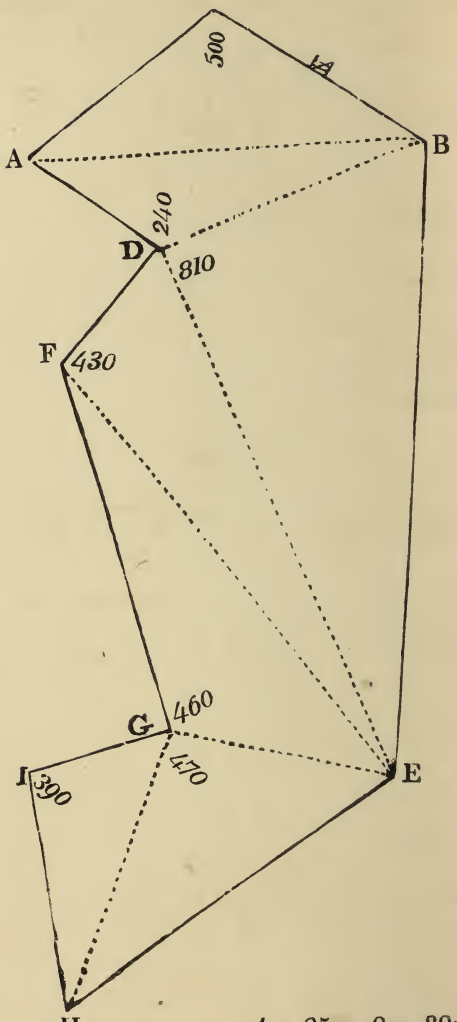


Ans. 7 *a.* 3 *r.* 21·424 *p.*

Draw a plan, and find the content, of a field, from the following notes:—

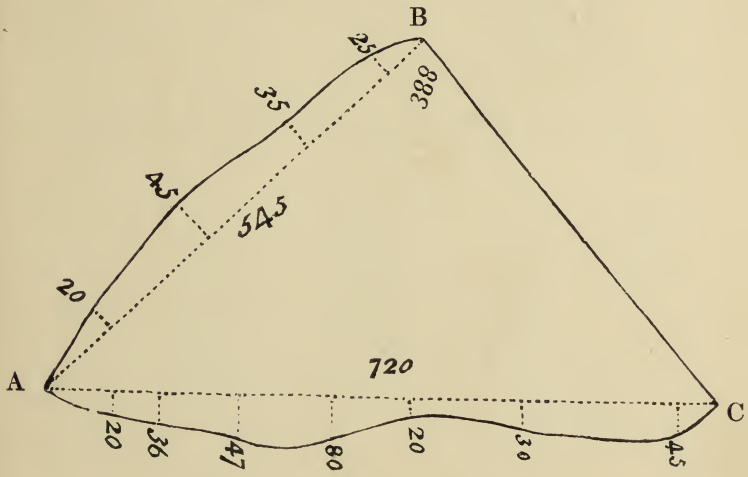
	444 I G R^{t} of I
	780 H I R^{t} of H
	1260 H E L of H
Diagonal	900 G H L of G
	1260 G F R^{t} of G
Diagonal	680 E G R^{t} of E
Diagonal	1670 F E L of F
	480 D F L of D
Diagonal	1826 E D R^{t} of E
	2000 B E R^{t} of B
Diagonal	890 D B L of D
	474 A D L of A
	760 C A L of C
	850 B C 440 L of B
Gate	
Diagonal Begin at	1260 A B A

Plan.



Ans. 25 *a.* 0 *r.* 39·184 *p.*

In measuring a line which has offsets on the left side, note them down in the left column of the field-book; if they are on the right side, note them in the right column.



Content 1 *a.* 3 *r.* 10·02 *p.*

In measuring from A to B, the notes will stand thus :

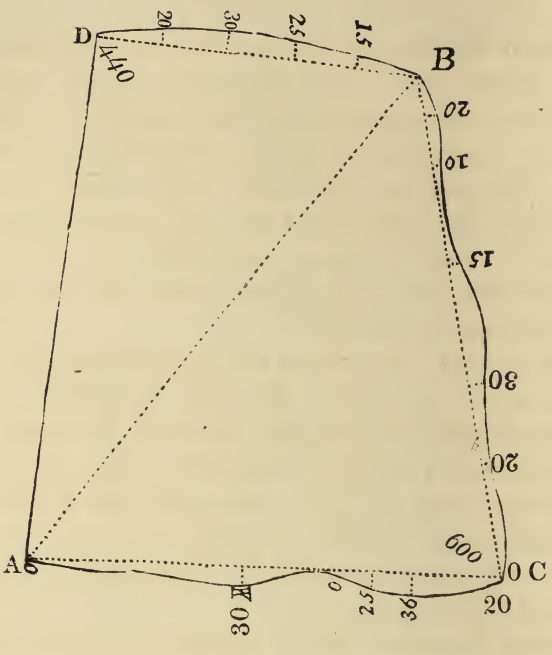
0	545 A B
25	500
35	380
45	240
20	100
Begin at	A

In measuring from A to C, the notes will stand thus :

	720 A C	0
	670	45
	500	30
	380	20
	300	80
	200	47
	120	36
	70	20
Begin at	A	0

Draw a plan and find the area of a field from the following notes:

	500 D B	0
	400	15
	300	25
	200	30
	100	20
	R^{t} of D	0
	843 A D	
	R^{t} of A	
	745 C A	0
	405	30 Gate
	260	0
	200	25
	140	36
	R^{t} of C	20
	808 B C	0
	630	20
	500	30
	300	15
	150	10
	55	20
	R^{t} of B	0
Diagonal	976 A B	
Begin at	A	



Content 5 *a.* 1 *r.* 30·592 *p.*

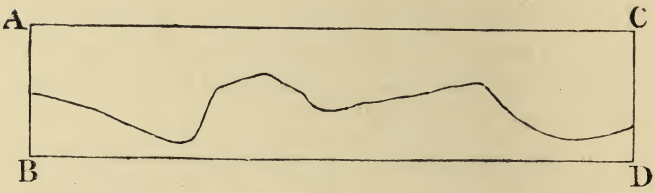
In surveying a farm or lordship, consisting of several fields, it is customary to draw a plan of the whole, and then compute the content of each piece separately by drawing fresh lines to divide them into trapeziums and triangles, the bases and perpendiculars of which must be accurately measured by the scale they were plotted from; in doing which, many of the crooked lines may be reduced to straight ones, by applying a straight piece of lantern-horn to the

crooked line, in such a manner that the small parts cut off from the crooked figure by it may be equal to those which are taken in, which equality of parts included and excluded, may be very nicely judged of by a little practice; then, with a pencil, draw a line by the straight edge of the horn. Or, instead of horn, use a bow made of wire and cane, which can easily be applied with one hand, and two dots made with the other to draw a line by.

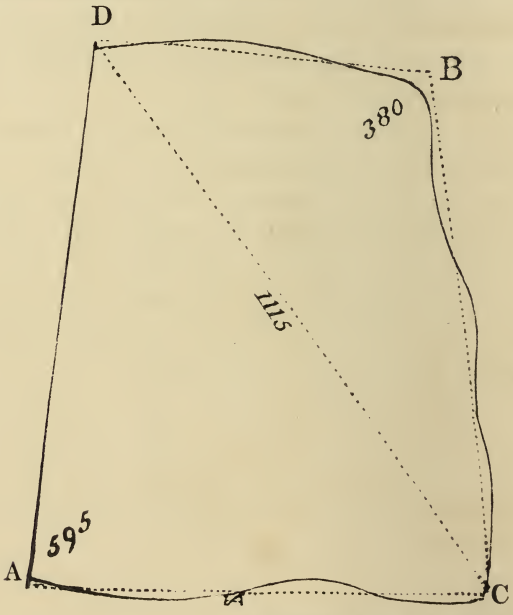
In this way, the work is very expeditiously done, and sufficiently correct; for such dimensions are taken as afford the most easy method of calculation; and amongst a number of parts thus taken and applied to a scale, it is likely some parts will be taken a small matter too little, and others too great, so that they will, upon the whole, in all probability, very nearly balance each other.

There is another method of managing offsets, by reducing them to one triangle with a parallel ruler, founded on the 31st Problem of Geometry, page 47: but, after the thorough investigation of it by every possible method, I find it tedious; and I have no hesitation in declaring, that straightening the crooked fences with a horn ruler is the most correct and expeditious of any method yet invented.

Find the area of the offsets on the line A C, by different methods, and of those on the line B D; add the two together, and find by which method their sum comes nearest the area of the parallelogram A B D C.



The following diagram represents the last field, with the fences straightened, &c., according to the foregoing method:—

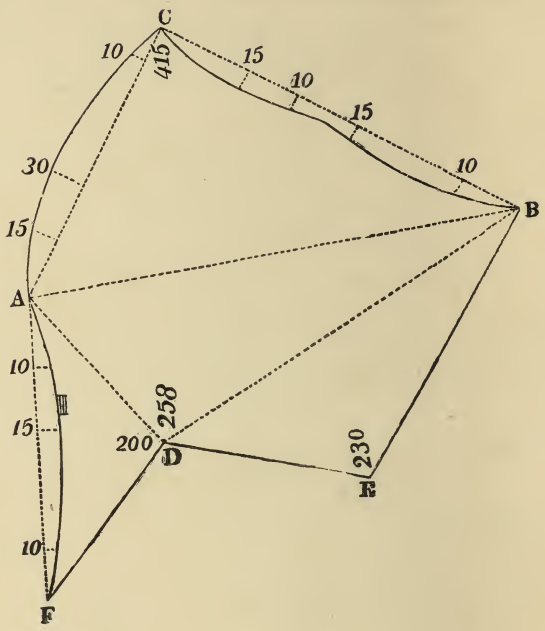


Content, 5 *a.* 1 *r.* 29·7 *p.*

Draw a plan, and find the area of a field, from the following notes:

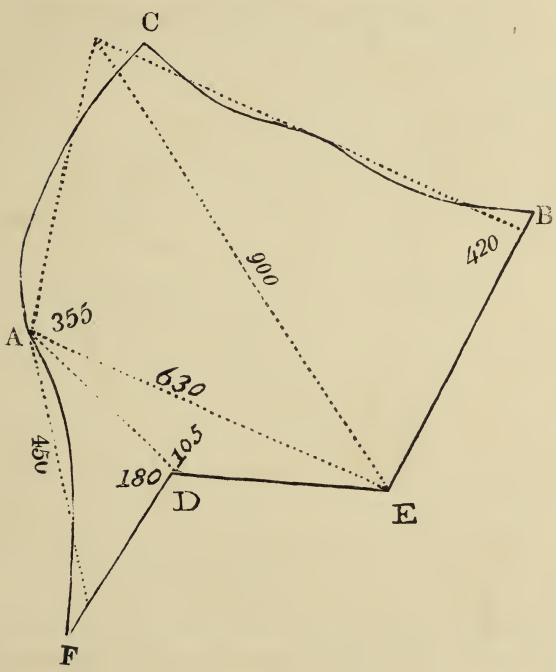
	500 F A	0
	400	30
	310	Gate
	300	40
	100	30
	R^{t} of F	0
	325 D F	
	L of D	
	345 E D	
	R^{t} of E	
	500 B E	
	R^{t} of B	
Diagonal	700 D B	
	L of D	
Diagonal	315 A D	
	L of A	
	505 C A	0
	400	50
	300	60
	60	30
	L of C	0
0	680 B C	
35	500	
20	410	
30	300	
20	100	
0	L of B	
Diagonal	825 A B	
Begin at	A	

Plan.



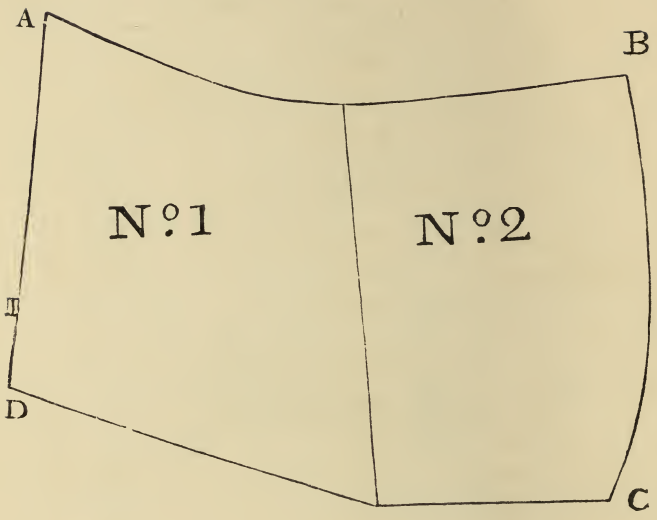
Content, 4 *a.* 1 *r.* 3·758 *p.*

Plan with the crooked Fence straightened.



Content, 4 *a.* 0 *r.* 35·72 *p.*

To survey two adjoining Fields together.



Proceed as if they were one field; in doing which, the fence which parts them will be crossed three times. Note down in what part of the chain-line the crossings are made, by drawing lines on that side the middle column the fence runs, or on both sides, if the fence runs on both sides. This will be easily understood by examining the following notes, and comparing them with the plan.

	1910 D C	0
straight ——	1145	
	L of D	150
	1050 A D	
	780	Gate
	L of A	
Diagonal	2260 C A	
straight ——	1000	—— straight
	R^{t} of C	
0	1350 B C	
75	1065	
100	760	
80	380	
	R of B	
	1860 A B	
	1470	60
	950	110 straight
0	360	
150	A	Begin

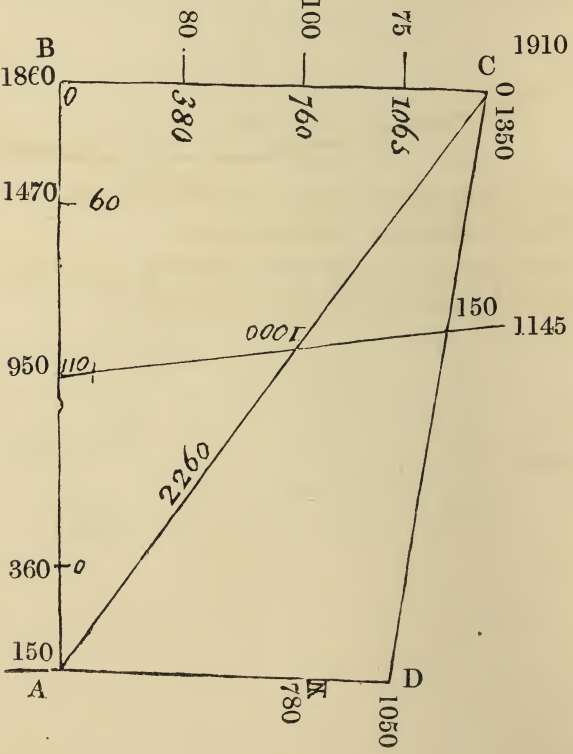
To plot them.

Draw the trapezium A B C D, and the offsets, as before directed. Mark the place on the diagonal where you crossed the fence, which stands thus in the notes:—

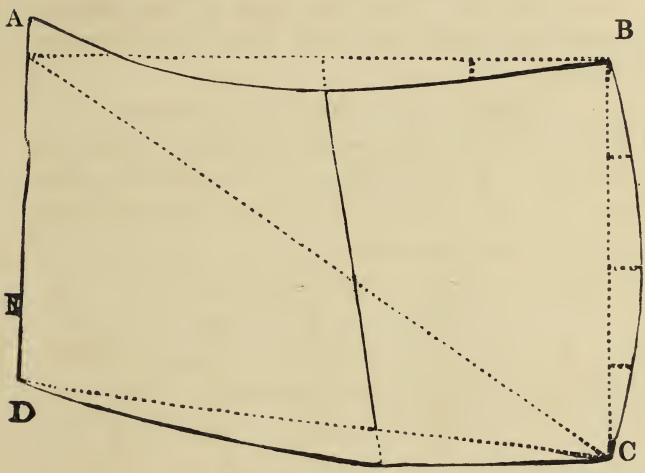
straight ——— 1000 ——— straight.

The words *straight* denote the fence crossed runs straight to the end; and, as it is crossed in more than one place, there is no occasion to measure it, unless to prove the work.

The plot will then appear thus:—

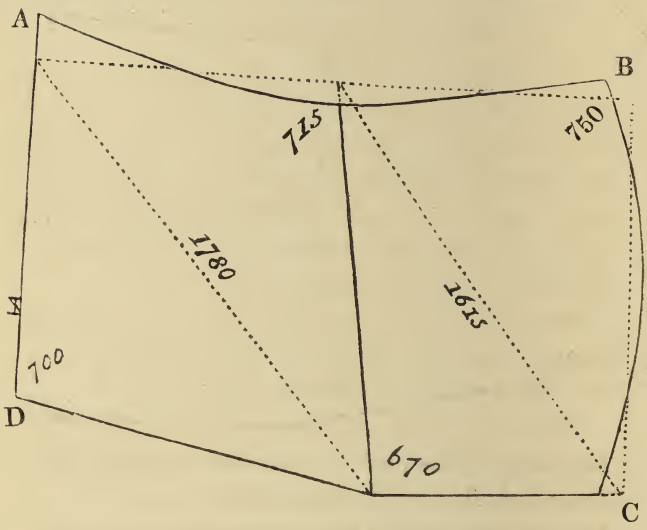


Draw the boundaries touching the extremities of the offsets, and it will stand thus:—



which, being cleared of the superfluous marks, &c., will be finished as in the first figure.

Find the area of each field separately, by straightening the fences, &c. as before directed.



Content of No. 1. - - 12 *a.* 2 *r.* 14·96 *p.*

Content of No. 2. - - 11 *a.* 1 *r.* 34·64 *p.*

PROMISCUOUS
OBSERVATIONS AND DIRECTIONS.

It must necessarily follow, the larger the number of fields, the greater will be the variety of memorandums and marks. It is impossible to invent a method that will suit all kinds of inclosures; but those who study these notes, and carefully lay down the pieces from them, may gain such a general knowledge of this method, as will enable them to survey by it.

Most surveyors keep their field-books in a similar manner; but each adopts his own marks, nor do any two follow the same method exactly.

When the learner attempts to survey several fields together, he must not burden his memory with too many lines at first. After a little practice, he may lay down the work twice a day; when more expert, once a day will do. He must not be discouraged if his lines will not prove: small variations will always occur with the most accurate; and a tyro will very frequently have to measure his lines twice. He must not be in haste to begin; but must walk over the tract of land two or three times, and observe how it can be divided into the largest triangles; for, in all surveying with the chain, whether of single or more fields, or of a parish, a chief point to be observed is, to measure the largest possible triangles which the case will admit. On the laying

down of which triangles the accuracy of almost every part must, in a great degree, depend.

In the following map, I have taken care to make the lines meet in the most convenient places, which I was enabled to do from the flatness of the land; but when the land is hilly, or the line so long that one end of a line cannot be seen when you are standing at the other, a cross will sometimes be found useful, somewhere between the two marks, to determine the direction of the line, by placing it in such a situation that both of them may be seen through one pair of sights. But when the two marks cannot thus be seen from one spot, observe the following directions:—Suppose A and B are two stations, between which the poles are to be ranged in a straight line, but in no part of it can both A and B be seen. Let one assistant place himself where he can see the mark at A, at as great a distance as he can from it, as at C.

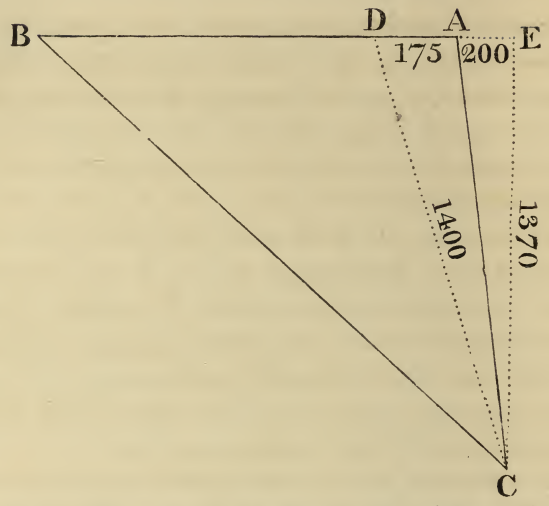
Let another assistant place himself somewhere between C and A, so as to have a sight of B, as at D. C and D must move themselves, till D ranges with A at C, and C ranges with B at D, which will be at *c* and *d*, where poles must be put up, and the lines may be measured.

The follower should have at least three poles before him, in a straight line. So soon as he comes up to the nearest, he must direct his assistant to get in a straight line with the other two, and fix another beyond the last; proceeding in this manner till the line is finished, which most likely will be either on the right or left of the mark measured for.

Or, fix all the poles up first, and then measure.

Suppose two lines A B and B C of the triangle A B C are measured, and, in measuring the third line, it ends at D, measure the distance from D to A: the notes will stand thus—

1400 C D L of C	175 to A



If the line ends at E, measure the distance from E to A, so that E B is one straight line: the notes will stand thus—

to A—— 200	1370 C E L of C	

It is best to dig holes of different shapes for marks, in which poles may be placed; if a pole be removed by accident, the hole will remain.

In taking small surveys, the different marks are easily remembered; but if the lines are of a considerable length, the shape of the mark dug in the ground must be entered in the field-book: thus—

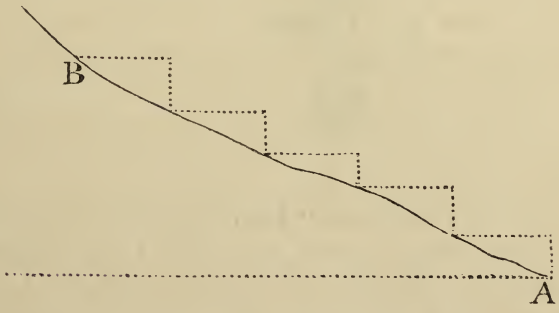
A △ B ▷ C ▽ D ◁ E ▷ F ⊾ G □ $_1$ = $_1$ I_0 K ⊓ L ⊃ M † &c.&c.

No poles should be left standing but such as serve for marks.

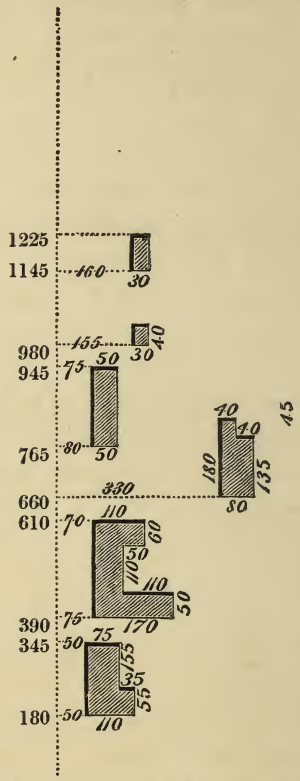
Many perplexing errors occur with young surveyors, from changing the arrows improperly. The leader must not ask for them till he wants one to put down, and has not got one: the leader and follower must then let the chain drop on the ground, meet each other half way, and each of them count there are ten; the leader must then return to his place, and put down an arrow. It is best to note down every change, thus: 1000, 2000, 3000, &c.; if this is neglected, in measuring long lines, an error of ten chains is very likely to happen.

If at any time an arrow is lost, return to the last mark, and measure over again.

In measuring rising ground, care must be taken to raise one end of the chain, so as to bring it to a horizontal direction; and, if the whole chain will not admit of an elevation sufficient, use only half the chain. In measuring from A to B, there will be an error of 60 links, if this method is not attended to.

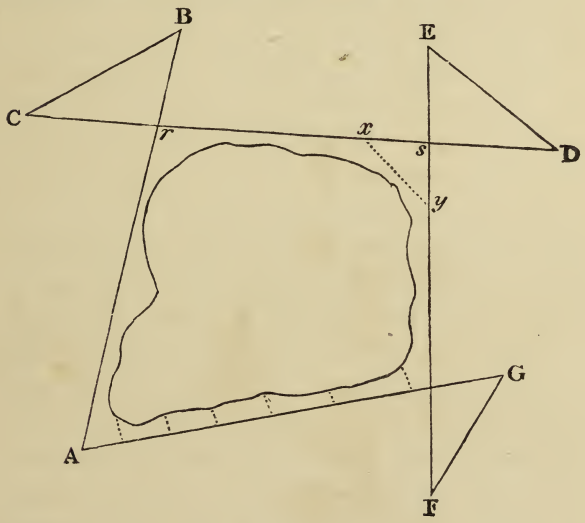


The following figures show in what manner the ground-plans of buildings are to be taken, which is too obvious to stand in need of any explanation :—



In order to survey a lake, wood, plantation, &c., without taking diagonal lines, observe the following directions:—Measure the two lines A B and C D, noting where they cross each other, and measure B C; then plot the triangle B C *r*, and produce the

line C *r* to D, and the line B *r* to A; thus will the two lines A B and C D be tied together, and the two *r* A and *r s* will be correctly plotted. In the same manner may the other lines be tied together, and offsets may be taken on every side, as on A G. From which plan the content may easily be found.



It is evident that, by measuring a line from a known point *x* in one leg C *s* of an angle C *s* F, to a known point *y* on the other leg *s* F, the lines C *s* and *s* F may be tied together. This will be found useful in taking an angle, when an instrument for that purpose is not to be had.

The distance of an inaccessible object may be measured with the chain, in the following manner:

Let A be the object, and the place of observation B; let an assistant go forward to C, and place a pole in a direct line with the object A; let B change his situation to *b*, and let C place a pole in a line between *b* and A, as at *c*; measure the four sides of the trapezium B C *c b*, and a diagonal *b* C, which must be accurately plotted. Produce the lines B C and *b c* till they intersect, which will be the proper situation for A, and the distance from B to A may be measured from the scale it was plotted from.

After having finished plotting the whole estate, the fences must be straightened, and the content of each field found as before directed. To prove the work, you must consider the whole estate as one large field, the fences of which may be straightened, and the area found, as in a smaller field; which area ought to be nearly the same as the sum of the areas of all the fields.

To make a clean copy of the plan, lay the rough plan upon the paper or vellum intended for the finished copy; press them close together by weights, to keep them as level as possible; then, with a needle, prick the extremities of the straight lines, and also as much of the curved and irregular ones as will enable you to draw them on the new plan; separate the papers, and trace the outlines with a fine pen and Indian-ink. Common ink (which is liable to run, when the plan is coloured) must never be used in planning. There are also several other methods of copying plans; as by tracing-paper, the method of squares, a copying-glass, and particularly by the pentagraph, an instrument equally as useful to the scholar as to the experienced surveyor, and by far the most useful of all others, where the lines are very irregular, or the plans to be copied require enlargement or diminution; the latter object, if the lines are straight, may also be conveniently effected by the proportional compasses. As a reference to these instruments is indispensable to an understanding of their use, the description of them is here omitted, since the first method here detailed is the most convenient for the school, or learner's practice. Let the north side of the plan (if convenient) be at the top of the paper.

After having completed the boundaries, put in the gates, remarkable trees, hedges, &c., shade the water, pits, hills, &c., with Indian-ink.

Should it be desirable to colour the plan, wash in the several fields with different tints of green, made

of liquid verdigris and gamboge; the former of which may be thus prepared:—"Simmer half an ounce of distilled verdigris in half a pint of white-wine vinegar slowly over a fire, for about half an hour, in an earthen pipkin; when cool, pour it gently off its sediment, for use." Water may be washed in with Prussian blue, or indigo. Roads, with a pale tint of umber, or raw T. sienna. The ground-plans of buildings with a pale tint of lake.

As it is obvious, to any person acquainted with colours, that general and abstract directions respecting their use can serve no practical purpose, without the illustration of example, a new and coloured plan is given in the present edition of the following estate, tastefully finished; thereby affording the student a fair specimen of the present mode of ornamenting and colouring plans: and, to make it as generally useful as possible, some things are inserted not adverted to in the field-book; such as the shadow, which is intended to represent an elevation or hill: some trees and clumps are also introduced in the meadow, adjoining the river; these are more commonly found in parks and pleasure-grounds: these, and the shadow in the arable-fields, may be left out in the student's plan, as not actually occurring in the present, but which should be introduced when necessary. It will be seen, that the finished plan, to make it better suit the size of the work than it has heretofore done, is upon a reduced scale, and not in strict unison with those which precede it. This is a circumstance which can occasion no real

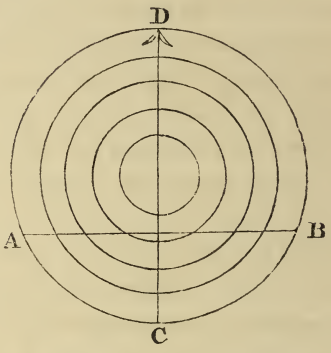
inconvenience; but the student should protract all his observations from one scale, that the steps of his plan may be the more distinctly marked.

To find a meridian line, take a compass somewhere in the middle of the estate, and observe in what direction the needle points; the true meridian-line will be $24\frac{1}{4}$ degrees to the east of the needle (that being nearly the present variation at London).

Another method.—Draw five or more concentric circles, about a foot asunder, on a level piece of ground, somewhere in the middle of the estate, and in the centre of them fix a pole perpendicularly, of such a height, that the whole of its shadow may fall within the circles from nine in the morning till three in the afternoon. Watch the extremity of its shortening shadow, in the forenoon, and mark where it touches the several circles; and, in the afternoon of the same day, watch the extremity of the lengthening shadow, and mark where it touches the circles. Draw a straight line A B, touching the two marks made on any *one* circle; bisect this line with another C D, drawn at right angles to it, which will pass through the centre, and be a true meridian: produce this line both ways till it touches some known boundary; it can then be plotted in the map, and, if it is not in a convenient place, it may be transferred to any other, by drawing another parallel to it.

The best time for drawing a meridian line in this manner is about the 21st of June, because the sun

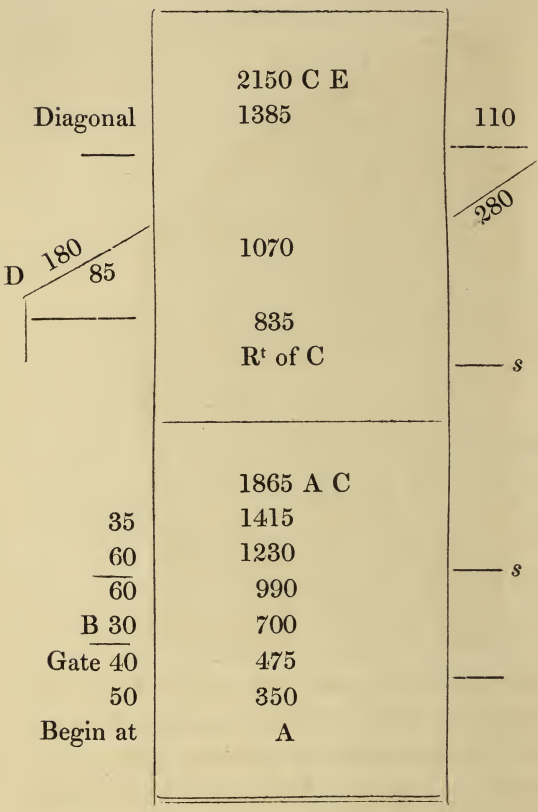
then changes its declination slowest, and its altitude quickest.



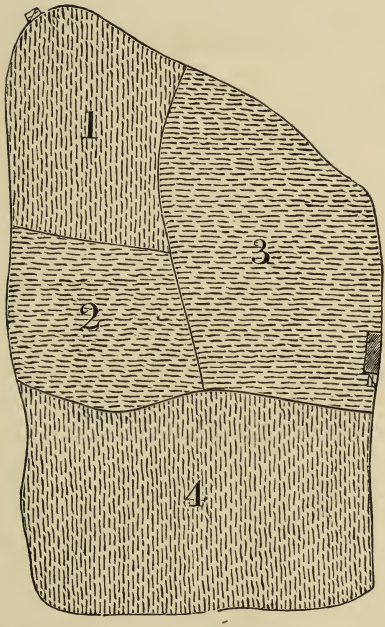
To survey four Fields together, the notes for which will be found in the following Field-book, beginning at bottom.

	FINIS.	
	610 I D	0
	500	30
	350	50
	250	40
	R^{t} of I	0
0	1120 F B	
50	840	
	620	0
	350	45
	L of F	

	1145 H G	
50	570	
40 | 1	400	
30	270	
0	R^{t} of H	
	200 C H	
Gate	100	
25	70	
	R^{t} of C	
Diagonal	1250 G C	
——	710	——
	L of G	
	1310 E G	
	1000	40
	850	40
		—— Barn
	720	40
	700	Gate
	570 F	10
	380	0
	L of E	
0	1080 E A	
35	700	
35	500	
35	270	
0	R^{t} of E	



Note.—*s* placed against a line denotes it is straight to the end.

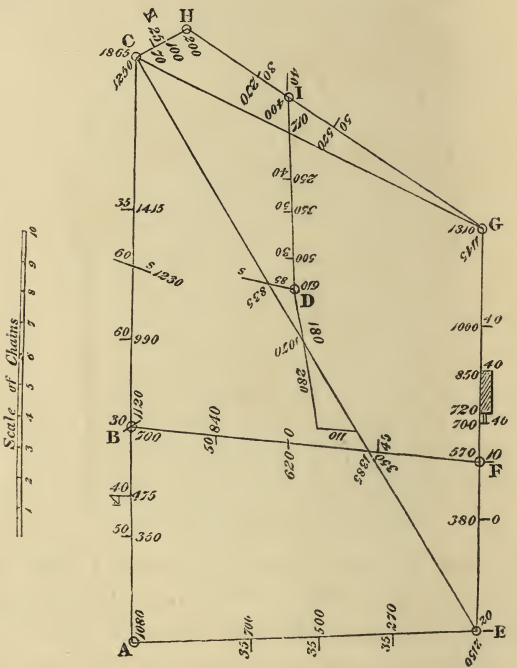


The first line is A C 1865, the second C E 2150, and the third E A 1080. With these three form the triangle A C E, and make the different marks on it, as in the following figure. With the next two lines, E G 1310, and G C 1250, complete another triangle C E G on the base C E, and draw the appendages thereto.

With the next two lines, C H 200 and H G 1145, complete the other triangle C H G.

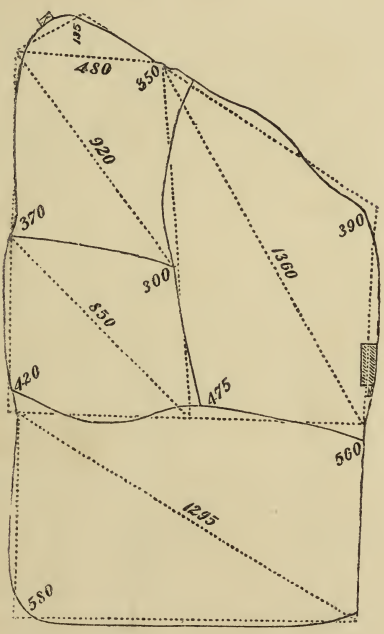
The remaining two lines, F B and I D, must be plotted, in order to show the internal fences.

When all the chain-lines, offsets, and other marks are drawn, the plan will appear as in the following figure:—



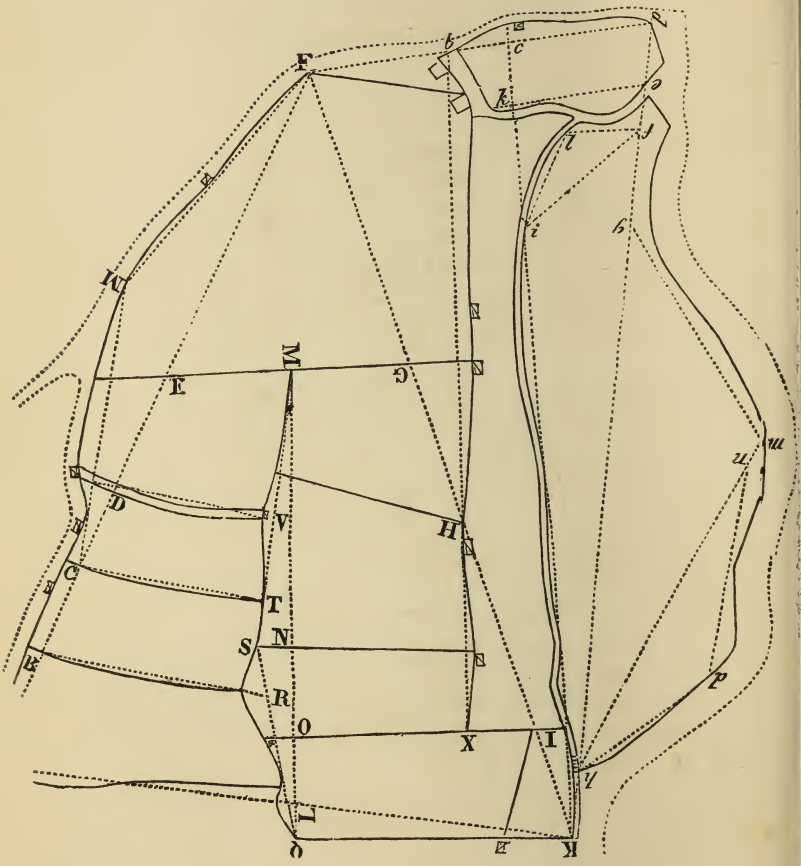
Draw the boundaries touching the extremities of the offsets, clear them of all superfluous marks, and the plan will be finished, as in the first figure.

The area of each piece is computed by straightening the crooked fences: thus—



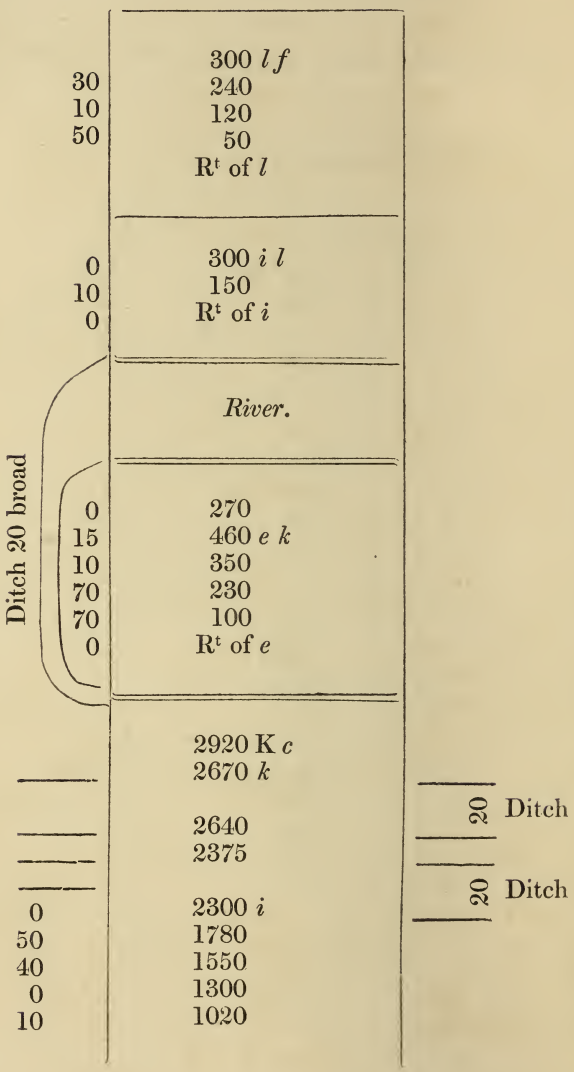
Content of No. 1. - -	3 *a.*	2 *r.*	21·76 *p.*
Content of No. 2. - -	3 *a.*	0 *r.*	9·6 *p.*
Content of No. 3. - -	5 *a.*	3 *r.*	21·12 *p.*
Content of No. 4. - -	7 *a.*	1 *r.*	21·04 *p.*

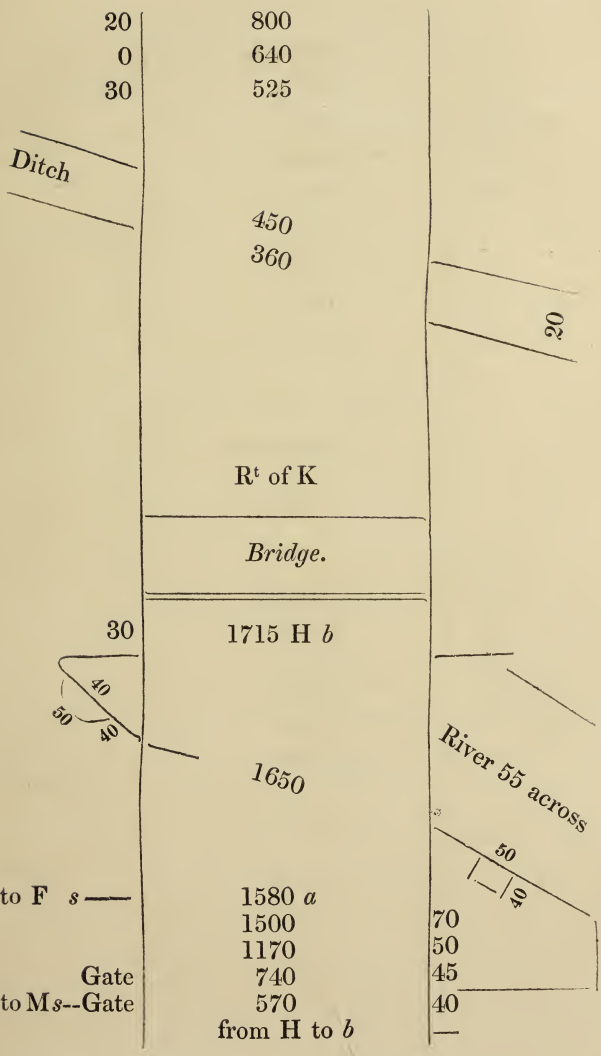
Directions for surveying and planning a small Farm.

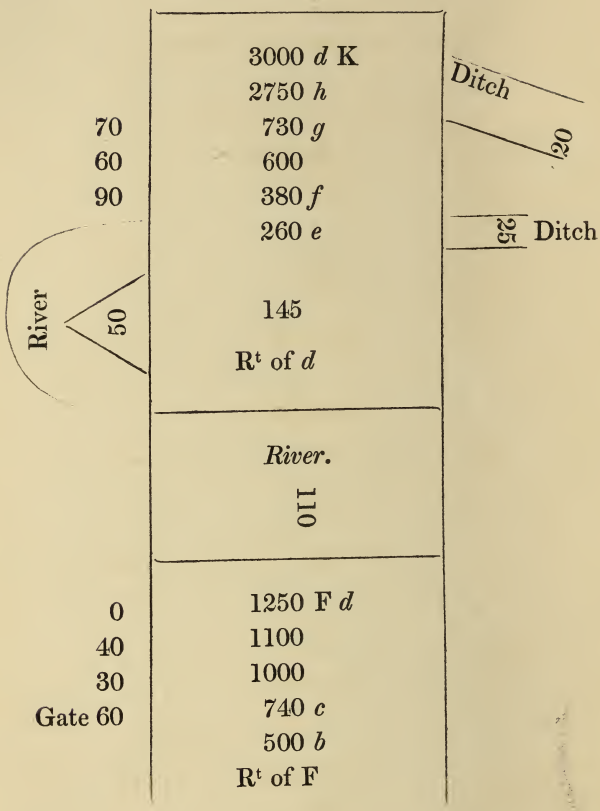


Notes for the Farm.

	840 *pn*	40
	700	50
	440	0
	100	60
	L of *p*	
	615 *hp*	10
	400	0
	100	20
	L of *h*	
	1420 *mh*	
	70 *n*	
0	R^{t} of *m*	
0	900 *gm*	
40	765	
50	525	
0	300	
70	L of *g*	
	520 *fi*	
	R^{t} of *f*	

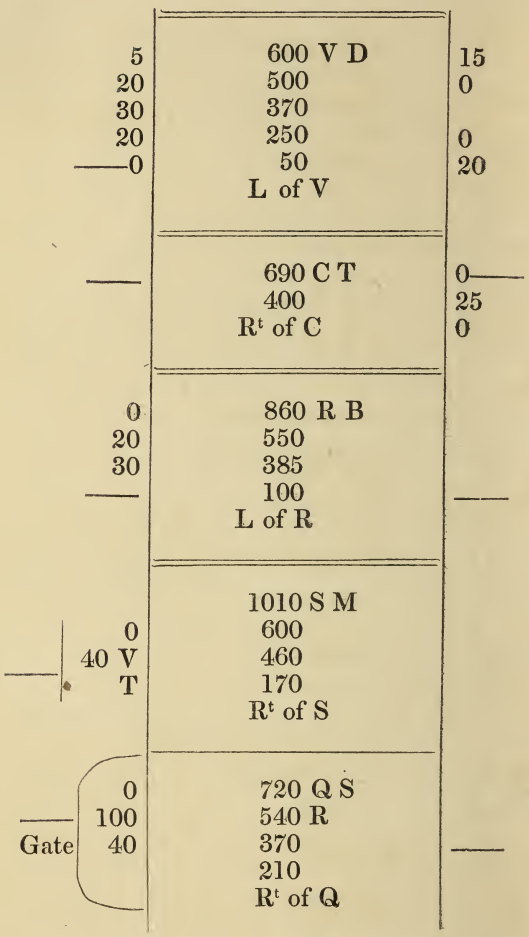

300 l f
30
240
10
120
50
50
Rt of l
300 i l
0
150
10
Rt of i
0
River.
Ditch 20 broad
270
0
460 e k
15
350
10
230
70
100
70
Rt of e
0
2920 K c
2670 k
2640
2375
2300 i
0
1780
50
1550
40
1300
0
1020
10
20 Ditch
20 Ditch

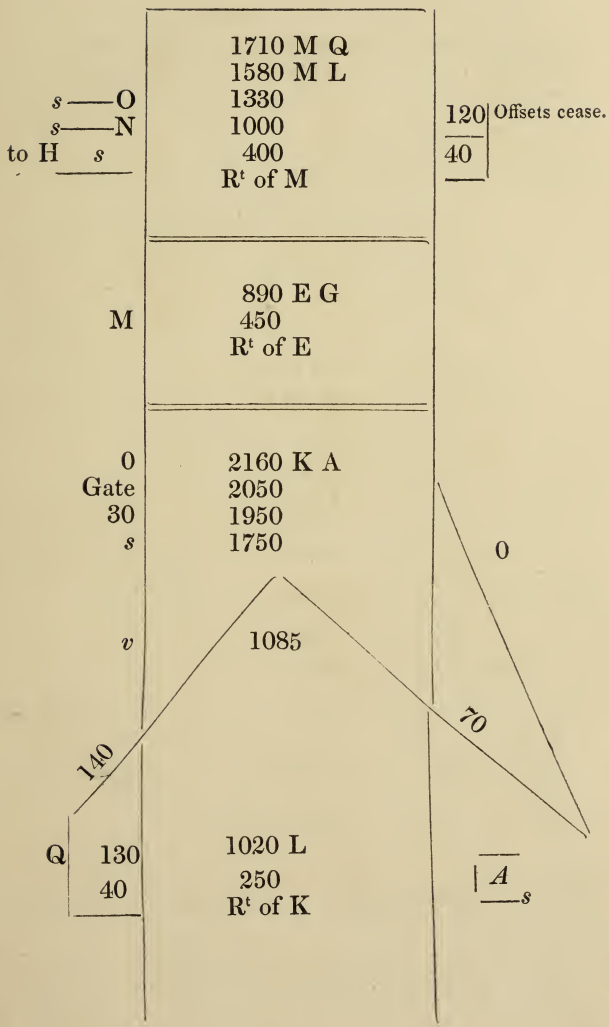

20
0
30
800
640
525
Ditch
450
360
20
Rt of K
Bridge.
30
1715 H b
40
50
40
River 55 across
1650
50
40
to F s
1580 a
1500
1170
70
50
Gate
740
45
to Ms--Gate
570
40
from H to b
—

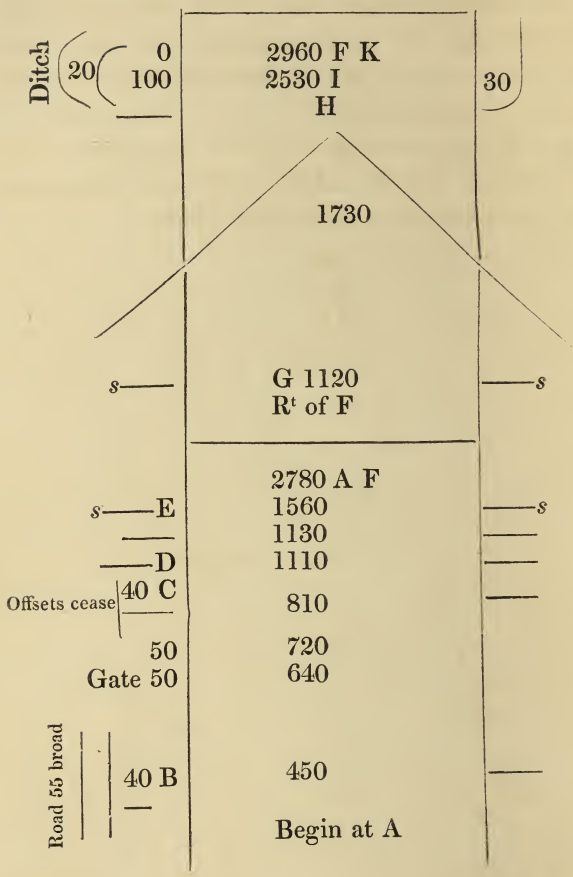

3000 d K
2750 h
70
730 g
20
Ditch
60
600
90
380 f
260 e
25
Ditch
River
50
145
Rt of d
River.
110
0
1250 F d
40
1100
30
1000
Gate 60
740 c
500 b
Rt of F

End of the first large Triangle.

0	1020 W F	
30	850	
0	450	Gate
30	150	
0	R^{t} of W	
0	1030 C W	
60	670	—
50	320	—
45	290	Path
0	170	
Gate	100	
40	L of C	
I 230 X	760 H X	—*s*
Gate	500	
40	470	—*s*
0	220	
Gate	50	
0	R^{t} of H	


5
20
30
20
0
600 V D
500
370
250
50
L of V
15
0
0
20
690 C T
400
Rt of C
0
25
0
0
20
30
860 R B
550
385
100
L of R
1010 S M
0
40 V
T
600
460
170
Rt of S
0
100
Gate 40
720 Q S
540 R
370
210
Rt of Q

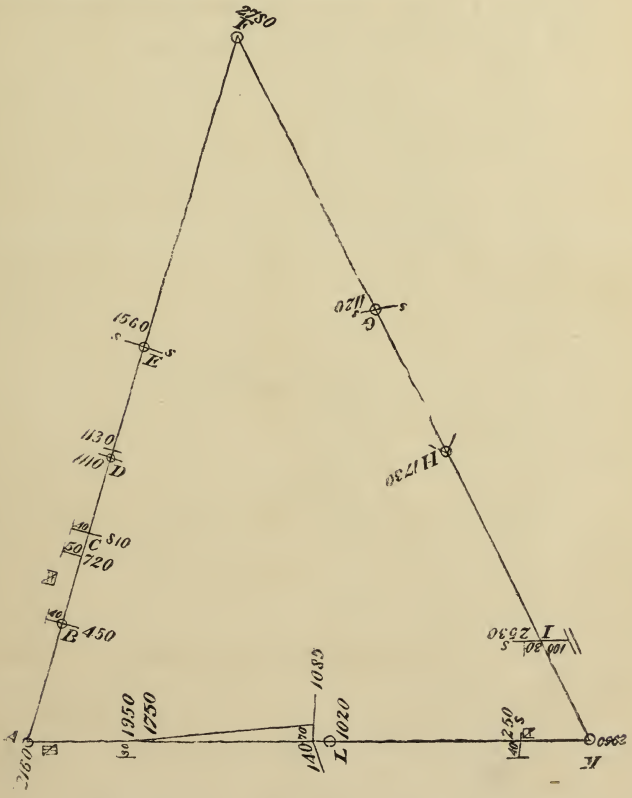

1710 M Q
1580 M L
s —— O
1330
s —— N
1000
to H s
400
Rt of M
120
40
Offsets cease.
890 E G
M
450
Rt of E
0
2160 K A
Gate
2050
30
1950
s
1750
0
v
1085
70
140
Q
130
1020 L
40
250
Rt of K
A
s



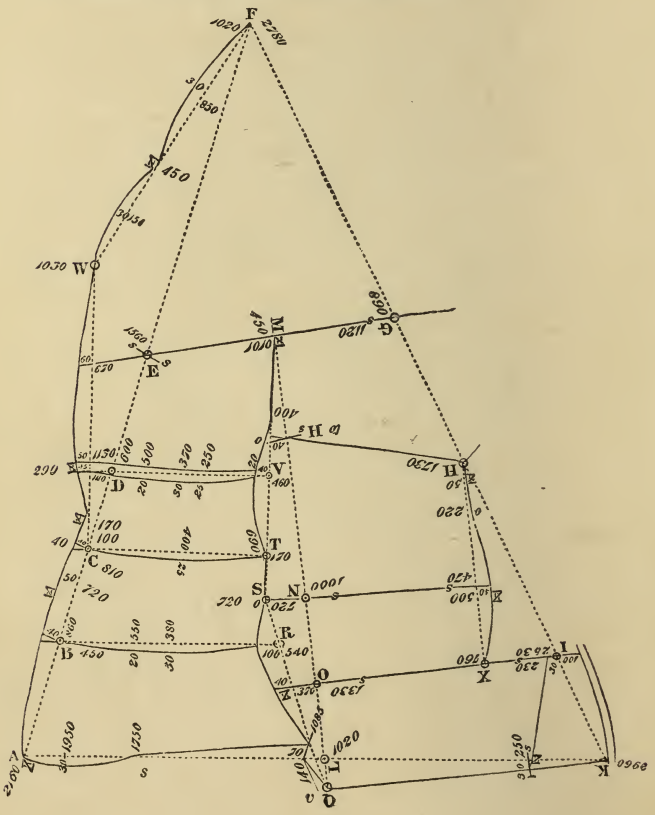
Note.—*s* placed against a line denotes it is straight to the end.

The straight dotted lines divide the whole into triangles, and are to be measured with the chain, the notes for which will be found in the field-book, beginning at bottom.

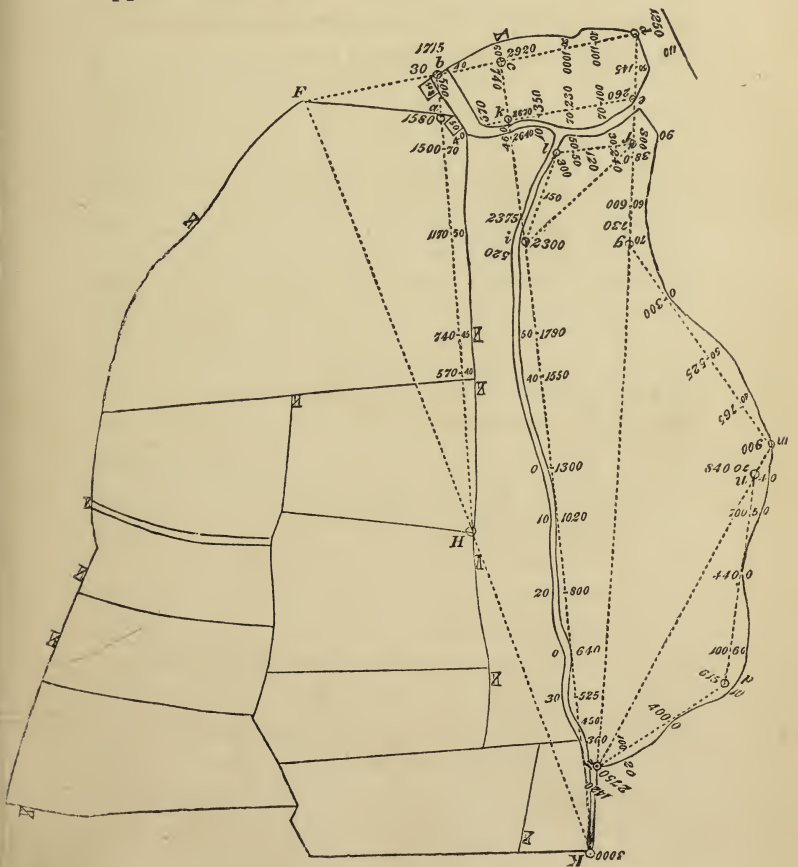
Construct a triangle with the first three lines, A F 2780, F K 2960, and K A 2160; draw the offsets and crossings, and it will appear thus —



Next, plot the lines within the triangles, *viz.* E G 890, M Q 1710, Q S 720; S M 1010, R B 860, C T 690, and H X 760; then draw the appendages belonging to them. The other lines run in a straight direction, and, being crossed in two places, need not be measured, unless to prove the work. Then plot the two lines C W 1030 and W F 1020, forming the triangle C F W. Draw the boundary-lines, and it will appear as in the following figure:—



Next with the two lines F *d* 1250, and *d* K 3000, complete the triangle F *d* K; and plot the internal lines H *b* 1715, K *c* 2920, *e k* 460, *i l* 300, *l f* 300, and *f i* 520. Then plot the two smaller triangles, *g m h* and *h p n*, with their proper offsets and crossings; draw the boundary lines; and the plan will appear thus:—



which being cleared of the superfluous marks, the fences must be straightened, and the content of each field found, as before directed; or draw perpendiculars to the triangles A F K, F *d* K, *g m h*, *n p h*, C W F, and find their areas, together with the offsets round them; the whole added together will be the content of the farm. When this is done, draw a fresh plan, and ornament it as the coloured plan.

The perpendiculars of the triangles A F K=1950, F *d* K=1210, *g m h*, = 550, *n p h* = 260, and C W F = 275 links, and may be measured as directed at *page* 136.

	Square Links.
The area of the triangle A F K	=2886000
The area of the triangle F *d* K	=1815000
The area of the triangle *g m h*.....	= 555500
The area of the triangle *n p h*	= 175500
The area of the triangle C W F.....	= 270875
The area of the offsets along *b d*.....	= 25400
The area of the offsets along *d e m p h*	= 98325
The area of the offsets along K *v* and *s* A ....=80825 The area of the offsets along *v s*, which deduct =23275	= 57550
The area of the offsets along A W F.....	= 78475
	The sum=5962625

Hence (5962625÷100000)×4×40=59 acres, 2 roods, 20·2 perches, the content of the whole farm.

PART THE SEVENTH.

TO SURVEY BY MEASURING THE ANGLES AND LINES.

However easy the theory may be, which teaches the survey of an estate by the help of the chain and cross-staff alone, the practical surveyor is too well aware of its difficulties in the field, often to venture upon it; he therefore usually provides himself with some instrument for the mensuration of angles, thereby facilitating his operations, and checking errors which may arise in the mensuration of long distances by the chain.

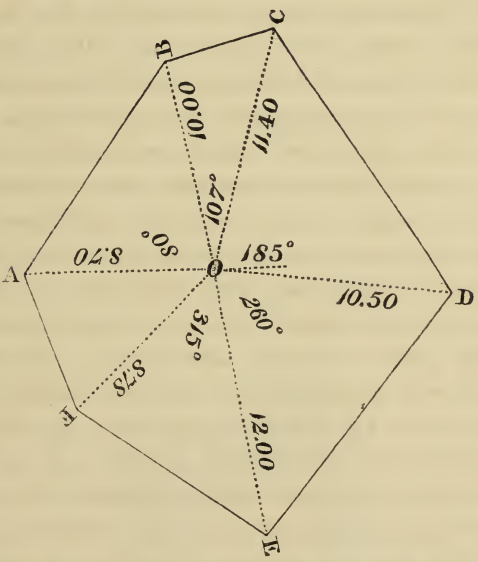
To any person moderately acquainted with trigonometry, the measure of one side of the preceding triangle with any two of its angles will render unnecessary the linear measure of its other sides—a very important object where lines are inaccessible: and where the lines are, or can be measured, the mensuration of an angle is very important for the verification of a triangle, upon the accuracy of which, perhaps, the whole survey may depend. Of all the various instruments for the mensuration of angles, the Theodolite is unquestionably the most important; but since, for ordinary and school pur-

poses, cheaper and more portable instruments have been invented, we shall confine ourselves to a more enlarged description of the improved Brass Pocket Box-Cross; and a still more useful and portable instrument, called the Pocket Sextant, with the improved adjustments made thereto by Mr. Banks, optician, Strand. As, however, the Plain Table is still much in use, and well calculated to bring theory and practice together, some description of it, and the method of its use, may be serviceable in this place, as any ingenious mechanic may construct it at a small expense. It is usually a level board of wood, like a common drawing-board, from twelve to fifteen inches long, and about ten or twelve broad, with or without a rim, for the purpose of keeping a sheet of paper tight upon it. On those constructed by mathematical instrument-makers are various scales for drawing parallel lines, as also the 360 degrees of a circle, for the estimation of angles; and on one side of the table a compass is usually fixed, for placing the instrument by; and the whole is fixed by a socket upon a three-legged staff, (of a suitable height for the eye), on which it is turned round, or fastened by a screw, as occasion may require. To the table necessarily belongs a thin flat narrow ruler, which, with two sights placed perpendicularly on its extremities, serves as an index: this ruler should also have a sloped edge, which is called the fiducial edge; and it is also usually graduated with various scales for laying down the lines; but

the necessity of this may be obviated by accompanying the instrument with a plotting scale.

Let a plan be required of a field of the following form and dimensions:

First, by the Plain Table.

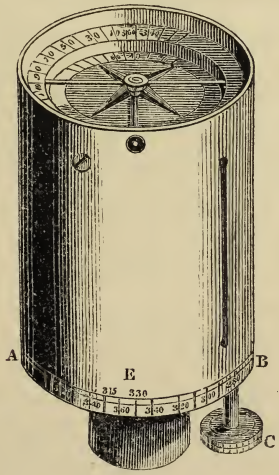


Place the instrument in any part of the field from whence all the angles may be seen, as at (O); fix it as horizontally as possible, and assume a point on the paper to represent the station of the instrument in the field; lay the edge of the index to this point, and successively direct it to the several angles of the field, A, B, C, D, &c., and draw indefinite lines by its edge towards the several angles OA, OB, OC, OD, &c.;

measure the several distances O A, O B, &c., and lay them off from any suitable scale upon their correspondent lines upon the paper; connect the several points by lines together, and a plan of the field will be obtained: if the boundary-lines are not right lines, offsets may be taken therefrom, as before taught, and irregular boundaries made. The plan of the field may also be taken by putting the instrument in either of the corners of it, and proceeding as before, or by successively planting the instrument in the several corners, and measuring round the field or wood, internally or externally, thus: Having placed the instrument as near as possible to the angle A, set up marks, at the same distance from B and F, as the instrument is situated from the angular point A: from the point assumed to represent upon the paper the station of the Plain Table, direct the index to the marks at B and F, and draw indefinite right lines in the direction thereof; from a suitable scale, lay off upon these the length of the lines A B and A F; then, removing the instrument to the mark at B, turn the Table, so that the index being upon the protracted line A B, from the point B (through the sights), a mark where the instrument stood at A may be seen; screw the instrument, that it may remain steady in this situation, and next direct the index to a mark equidistant, as before, from C; measure B C, and lay it down in the given direction; next plant the instrument in C, &c., and proceed as before, till the plan be completed. In a manner analogous to this

method of planning the field, may the paper itself be shifted, when various plots are found to exceed its dimensions.

Among the various instruments for measuring angles, one of the best for the use of school-boys is the improved Brass Pocket Box-Cross, because it can either be used as a common Theodolite, Circumferentor, or Cross-staff; and it costs less than other instruments which are commonly used for the same purpose. The graduated base A B, is the circumference of a circle divided into 360 degrees, which being screwed to the staff, and the staff fixed firmly in the ground, the other upper part of the instrument may be turned either way by rack-work and pinion at C.



To measure an Angle, A B C.—*See the Diagram at page* 231.

Place the instrument over the angular point B; move the pinion C till the Index of the Nonius E coincides with the 360th degree. Look through the sight which is directly above the Index E, and turn the instrument on the staff till you can see one of the marks, for example, A; then screw the instrument fast to the staff by the tightening-screw; turn the pinion C to the right, till you can see the right-hand mark C through the same sights; the Index of the Nonius will point out the number of degrees; and the coincidence of a division on the Nonius with one on the circle below, will show the number of additional minutes (if there are any) which the angle contains.

Explanation of the Nonius.

The moveable scale E, immediately above the graduated base A B, is called the Nonius, or Vernier scale, and is so contrived that the divisions of the base, which are each equal to one degree, is subdivided thereby into twentieths, so that the measure of the observed angle may be obtained to three minutes of a degree.

To illustrate the use of this scale, let two cases be supposed:

First. When the Index of the Nonius exactly coincides with any division of the base, that degree against which it stands is the measure of the required angle;

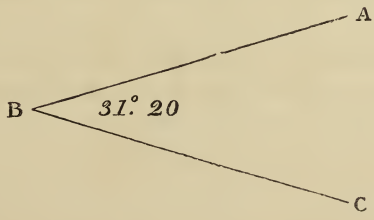
thus, let the Index (which is plainly distinguished upon the Nonius) stand against 30° on the base, the measure of the angle is 30°.

Secondly. If the Index does not exactly coincide with any particular degree upon the base, some one division of the Nonius will; observe which it is; and that division will give the odd minutes to be added to the degree pointed out by the Index division; thus, suppose the Index points between the 32d and 33d degrees, and that division of the Nonius which coincides with a division of the base is that marked 21, then the required angle will be 32° 21′.

Observe, in this instrument each division, as before observed, represents three minutes.

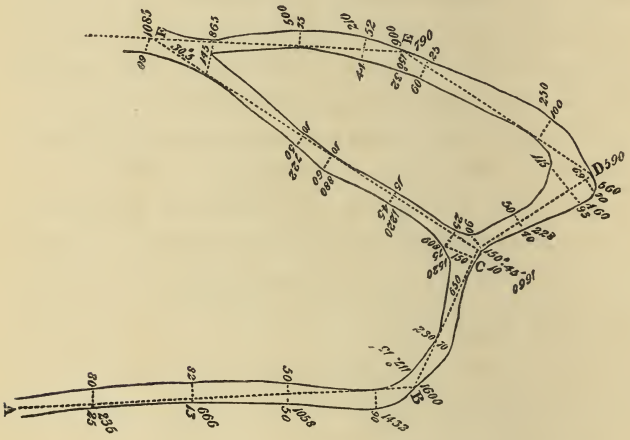
In Theodolites, angles can usually be obtained to a minute of a degree, and sometimes to less. A careful inspection will soon teach the method of reading the required angle off.

To plot an Angle.



Draw the line A B; place the edge of a Protractor on it, so that the centre point coincides with the

angular point B; then by the graduated edge prick off the number of degrees; and a quarter, or half, or three-quarters of a degree may be pricked off with a fine needle by inspection; but it is best to have a circular Protractor, with a moveable Index and Nonius, by which the number of minutes may be pricked off with the accuracy of three, or even one, minute.



In surveying the roads and field, by measuring the angles, &c., the notes in the field-book, for the preceding figure, will stand thus :—

	End.	
	F C 1660	
25	1520	75
15	1220	45
10	880	60
10	722	50
	L of F	60
	30° 5′ ⟩	
	E F 1085	
	865	
	500	75
44	210	52
	L of E	
	150° 32′ ⟩	
	D E 900	
60	790	25
	250	100
	L of D	
	69° 37′ ⟩	
	C D 590	
	560	70
115	460	93
50	228	40
90	R^{t} of C	
	150° 45′	
	B C 650	
150	608	
	230	70
	L of B	
	117° 15′ ⟩	
	A B 1600	0
0	1432	90
50	1058	50
82	666	13
80	235	25
	Begin at A	

THE POCKET SEXTANT,

With improved Adjustments.

Fig. 1.

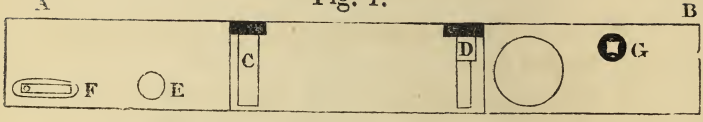
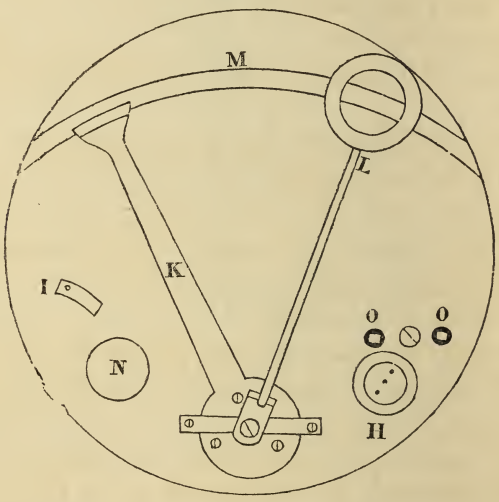


Fig. 2.



THE Pocket Sextant is, in many instances, more useful than the Pocket Box-Cross. In the first place, it requires no stand to fix it in the ground, but is used in the hand; it will also serve as a substitute

for Hadley's quadrant, in taking altitudes, &c., as well as measuring distances. It is necessary in this Treatise to explain its uses in Land Surveying only.

An Explanation of the different Parts of the Instrument.

Fig. 1. A B represents the circular rim extended to a straight line.

C is the index-glass.

D is the horizon-glass.

E is the sight-hole.

F is a screen-pin.

G is the adjusting-screw, to rectify the index error.

Fig. 2 represents the face of the instrument.

H is the index-pinion.

I is the screen-pin.

K is the index, at the end of which is a nonius.

L is a microscope.

M is the graduated arc.

N is the key, that unscrews, and fits O and G.

O O are the adjusting screws, to set the horizon-glass perpendicular.

By turning the index-pinion, the position of the index-glass may be altered, and, at the same time, the index made to point to any degree and minute on the arc.

The sight-hole directs the eye to the horizon-glass.

By moving the pin F, a screen-glass is brought before the sight-hole, to counteract the intense rays

of the sun, when the instrument is used for astronomical purposes.

By moving the other pin I, another screen-glass is brought to intervene between the sight-hole and the horizon-glass.

The divisions on the arc and nonius are too minute to be observed by the naked eye, but they may be accurately read with the microscope.

However accurately the instrument may be formed, the horizon-glass is liable to error, but may be rectified by turning the adjusting-screw at G with the key.

The index-glass C is a flat square piece of glass, truly ground and polished, quicksilvered on the back, and fixed in a brass frame. The use of this mirror is to receive the rays from an observed object, and reflect them on the horizon-glass.

The horizon-glass D has its upper half quicksilvered, and the lower half left transparent, so that an observed object may be seen through the transparent part, and the reflection of the same object in the silvered part.

To rectify the Instrument.

Previous to using the instrument, it must be properly adjusted; to do which, set the index on the nonius exactly at O on the arc, place a pole vertically at any convenient distance, look through the sight-hole, and hold the instrument in such a position as to see the pole through the transparent part

of the horizon-glass; you will at the same time see it reflected on the silvered part of it; if the reflected part of the object exactly coincides with the real object, so that the two parts appear as one whole, the instrument is correct; but if the reflected part does not coincide with the other part, it must be brought to coincide by applying the key to the screw G, and turning it backward or forward till it is correct. Next, take for an object the top of a hedge or a wall, or any other horizontal line; and if that line should, both in the quicksilvered part and the transparent part, be seen to coincide, the instrument is correct in that part: but if the line is seen double, the key must be applied to the adjusting lever O, and moved till they do coincide.

To measure the Angle A B C, *Page* 231.

Set up marks at A and C, and take your station at B.

Set the index of the nonius at O on the arc. Look through the sight-hole, and find the right-hand object C, which will be seen in the transparent and reflecting part of the horizon-glass. Turn the index-pinion toward the left-hand mark A, and at the same time move the instrument so as to retain the reflection of the first object in the horizon-glass, till you see the object at A through the transparent part of it, and make the two objects to coincide. The nonius will show the numbers of degrees and minutes contained in the angle. An angle may be accurately measured,

although the instrument is not truly adjusted, by observing the index error.

To find the Index Error.

Set the index at O, and if the objects do not coincide, turn the index-pinion till they do coincide; observe, by the index, the amount of the error. If the nonius stands between O and 99 degrees on the arc, then the error is to be subtracted from the angle; but if it stands on the other side of O on the arc, the error is to be added. No instrument can be more convenient or expeditious than the Sextant, for the setting off offsets. Adjust the instrument, and set the index to 90 degrees; walk along the station-line with the instrument in your hand, always directing the sight to the farther station-staff; let the assistant walk along the boundary-line; then, if you wish to make an offset from a given point in the station-line, stop at that place, and wait till you see your assistant by reflection; he is then at the point in the boundary through which that offset passes. On the other hand, if you wish an offset from a given point or bend in the boundary, let the assistant stop at that place, and do you walk on in the station-line till you see the assistant by reflection in the instrument, and that will be the point where an offset from the proposed point or bend will fall.

The explanation of the use of the nonius, and the method of surveying by measuring the angles, has been already explained.

USE OF THE THEODOLITE.

To take the Plan of a Field from any station within it, from which all the Angles may be seen by the Theodolite. See Figure, (page 227.)

1st. PLACE the instrument in any part, as at (O); then spread the legs of the Theodolite sufficiently wide, and thrust them far enough into the ground to keep the instrument steady. 2dly. Set the instrument horizontal, which may be accurately done, if furnished with a level. 3dly. Screw the ball firmly in the socket, that, in turning the index, the Theodolite may not vary from the objects to which it is directed. Next direct the fixed sights along the line OA, so that through the same the mark at A may be seen; screw the instrument fast; then turn the moveable index, till through its sights the mark in the angle B may be seen; then the degrees cut by the index upon the graduated limb will be the measure of the angle AOC, which let be 80°; the instrument still remaining steady, direct the index as before, till through the moveable sights the object at C can be seen, when the angle AOC=107 will be obtained: proceed in the same manner till the several angles subtended at the instrument are obtained,

viz. AOD=185°, AOE=260°, and AOF=315°; next direct the index till through its sights the object at A can be seen; when, if the degree cut thereby be 360°, a proof is afforded that the instrument has remained steady during the operation. If the Box Sextant be employed, the angles AOB=80°, BOC=27°, COD=78°, DOE=85°, and EOF=55°, will be the angles obtained thereby: lastly, measure the several distances OA=870, OB=1000, OC=1140, OD=1050, OE=1200, OF=878 links, and the field-operations will be completed. The method of drawing the plan from the preceding particulars will be easy enough to any person understanding the method of laying off angles and lines, already explained in this work. The preceding plan might have been taken by placing the instrument in either angle of the field, taking the angles which each side subtended at the instrument, and measuring therefrom to the several angles; or by placing the instrument successively in each angle, and taking the angles of the field formed by its several sides with each other, and measuring round the field.

Examples for Practice.

1. Required the plot of a field, of the same number of sides as the preceding; the angles as follow: AOB=74° 20′, BOC=25° 15′, COD=86° 40′, DOE=93°, EOF=68° 30′: the sides, AO=459, BO=730, CO=821, DO=846, EO=954, FO=568 links.

2. Let the following angles be taken from the angle

A, in a field of the same number of sides with the foregoing: BAC=21° 10′, BAD=72° 40′, BAE=84° 10′, BAF=142°: the internal lines, BA=740, CA=830, DA=896, EA=763, and FA=490. Required the plan of the field therefrom.

3. Required the plot of a field, by measuring round the same, and taking the angles which the side subtend at the eye, the instrument being placed in one of the angles of the field, from whence all the other angles can be seen. Let the field be quadrangular, as the figure, *page* 125. The angle which AC subtends at the instrument at D=37° 10′, CB =58° 30′; the side, AD=571 chains, AC=224, CB =492.

Method of protracting the preceding Example.

Draw the right line AD at pleasure; assume any point therein for the place of the instrument, as D; place the centre of the protractor at D, and lay off the angles ADC=37° 10′, and CDB=58° 30′; draw indefinite lines from D through C and B. Upon DA lay off 571, from any suitable scale which will determine the angle D; with the distance AC in the compasses, and one foot in A, cross DC, which will determine the place of angle C in the plan: proceed in the same manner to find B; lastly, join BD.

Note.—The conformity of the protracted length of BD with the measured length will determine the accuracy of the survey and plan. It must also here be observed, in protracting the observations, an

ambiguity may arise in the proper intersection of the lines AC and CB with the indefinite lines drawn from D through those points, since the sides of the field, when taken in the compasses, will intersect the latter lines in two places, making the angles at C and B either obtuse or acute: a recollection, however, of the field will remove this uncertainty.

4. Required the plan of a field, similar to figure *page* 138, taken by the Theodolite, placed in angle A, and measuring the lines AC=390, AB=705, AD=950, and AE=565; angle, CAB=59° 30′, CAD=86° 45′, CAE=130°.

5. Required the plan of a field, similar to the figure, *page* 178, the sides as there given, and the angles of the field as follow: CAF=79°, AFD=142° 30′, FDE=76° 15′, and DEB=60°.

MISCELLANEOUS DIRECTIONS.

As it is highly desirable that works of public instruction should be practically useful, the Editor, to obviate as much as possible a common complaint often urged against works on Surveying, "that they do not contain sufficient directions to the young surveyor about to reduce his theory to practice," inserts, with great pleasure, the following directions; which he trusts will be found of great practical

utility, and for which he is indebted to his friend, Mr. J. Butler, Land-Surveyor, Cranbrook.

Items necessary to a Survey, Plan, and Terrier of an Estate.

1. That it should set forth in what parishes it lies, and also the boundary-lines of such parishes should be traced on the plan.

2. That the boundary of the estate, and also each piece, be exactly traced and laid down. In reducing the plan for portable use, care must be taken that the fields are sufficiently large to admit of their respective indentations; by which, questions relating to trees, fences, &c., may be determined.

3. That some device be adopted, to show to whom any particular fence or boundary may belong, for the purpose of its maintenance.

4. That the lines intended for the boundaries should be from dimensions taken according to the local custom, *e. g.* to the back stem of the hedge, where the fence should be maintained, and to the outside of the ditch, where both hedge and ditch constitute the boundary: that such boundaries should be drawn where they actually are, if within thirty feet of the middle of a turnpike-road, or fifteen feet of a common highway; but, if not within these distances, they should be drawn where they might be set out to, and may be deemed effective land. All boundaries without such distances should be deemed

half-roads, where the owner has a right to the soil, but not to inclose.

5. That the internal lines of divisions should be the mid stem of the hedge; and from each side should be deducted the rough or waste land (neither employing the plough or sickle), and which should be thus accurately designed in the terrier, in which also the quantity of titheable land should be entered, as in the following example :—

Number on the Plan.	Names of fields.	Cultivation.	Plough and Sickle.	Rough or uncultivated.	Entire Quantity.
	House		*a. r. p.*	*a. r. p.*	*a. r. p.*
No. 371.	Meadow.	Meadow.	2. 0. 30.	0. 0. 25.	2. 1. 15.

In the last two columns should be comprised sites of buildings, yards, gardens, ponds, inland-roads, rivers, hedges, ditches, belts of wood, underwoods, &c. Great care should be taken not to over-rate the plain-land, since for this the farmer pays, in lieu of being tithed in kind, labourers for workmanship, and estimates thereby the quantity of seed necessary to be sown, &c. &c.

6. That all underwoods should have three several quantities entered; the effective quantity (or that which is charged to the purchaser) being measured to the near stem of the hedge; the content of the hedge and ditch; and, lastly, the entire quantity.

7. That, in a finished plan, all underwoods, and belts of wood of a certain breadth, should be accu-

rately distinguished by colour or otherwise; care being also taken to distinguish therein such plots as are bare, or thin in wood.

8. That all principal roads, though private, through the estate, from one highway to another, should be laid down.

9. That all ancient highways, though now disused, that go through or skirt the estate, should be carefully noticed in the references, to show in whom the right of the soil is vested, and the privilege of re-opening them if necessary.

10. That all privileged or other roads through adjoining lands should be noticed.

11. That all commons of pasture, whether inclosed by mutual consent or open, should be accurately noticed, to distinguish the freehold from what is only appertaining.

12. That all highways, called footpaths or bridle-roads, through the estate, should be noticed, to determine the public right, and to guard against innovations.

13. That the gates and bar-ways, which give entrance into the several fields, &c., be noticed, both from off the highways and from one another.

14. That the proprietors' names of the adjoining lands be inserted, to corroborate the title.

15. That in giving the cultivation of each piece, great care should be taken to note which are per-

manent meadows, or such as the tenant is interdicted from ploughing, and also which are fruit-orchards.

16. That all detached parcels of land should have the way therefrom laid down, till it joins some public highway.

17. That the ancient and proper names of the fields be preserved, and no new ones assigned without sufficient warrant.

18. That the ichnography of all buildings should be laid down, as also the yards and gardens adjoining.

19. That the plan, if extensive, should have an index-character, as well as a numerical designation, for the more readily finding any particular farm.

QUESTIONS FOR EXERCISE.

1. The area of a rectangular field is 14 *a.* 2 *r.* 11 *p.*; what is its length, its breadth being 925 links? *Ans.* 1575 links.

2. A ladder, 40 feet long, may be so placed, as to reach a window 33 feet from the ground on one side of the street; and by only turning it over, without moving the foot out of its place, it will do the same by a window 21 feet high on the other side; required the breadth of the street. *Ans.* 56·6.

3. The perambulator, or surveying-wheel, is so contrived as to turn just twice in the length of a pole, or $16\frac{1}{2}$ feet; required the diameter. *Ans.* 2·626 feet.

4. If the diagonal of a square be 67 yards, what is the length of the side? *Ans.* 47·376 yards.

5. The area of a rectangular field is $7\frac{1}{2}$ acres, and the length of the diagonals 50 perches; required the sides.

Ans. 30 and 40 perches.

6. The longer diameter of an ellipse is 81 yards, and the shorter diameter 64 yards; what is the diameter of a circle of equal superficies? *Ans.* 72 yards.

7. A rectangular plantation of 360 acres contains 435600 trees; required the distance of the trees. *Ans.* 6 feet.

8. How many trees can be planted in 360 acres, at the distance of 12 feet? *Ans.* 108900.

9. A square within a circle contains 16 square yards; required the area of a square circumscribed about the same circle.

Ans. 32 square yards.

10. If my court-yard be 47 feet 9 inches square, and I have laid a

footpath with Purbeck-stone, of 4 feet wide, along one side of it; what will paving the rest with flints come to, at 6*d.* per square yard?

Ans. 5*l.* 16*s.* 0½*d.*

11. The paving of a triangular court, at 18*d.* per square foot, came to 100*l.*; the longest of the three sides was 88 feet; required the sum of the other two equal sides. *Ans.* 106·85 feet.

12. What is the side of that equilateral triangle, whose area cost as much paving, at 8*d.* a foot, as the pallisading the three sides did at a guinea a yard? *Ans.* 72·746 feet.

13. Given two sides of an obtuse-angled triangle, which are 20 and 40 poles, required the third side, that the triangle may contain just an acre of land. *Ans.* 58·876, or 23·099 poles.

14. Required the side of an equilateral triangle, whose area is just two acres. *Ans.* 6·79617 chains.

15. The sides of a triangle are 20, 30, and 40 respectively; what is the area of the inscribed circle? *Ans.* 130·8999.

16. In an isoceles triangle two circles are inscribed, touching each other and the sides of the triangle; the diameters of the circles are 9 and 25; required the sides of the triangle.

Ans. 44·27083, 44·27083, and 41·6′.

17. The area of a right-angled triangle is 60, and the hypothenuse 17; required the two legs. *Ans.* 15 and 8.

18. The base of a field, in the form of a trapezoid, is 30, and the two perpendiculars are 28 and 16 chains respectively; it is required to divide it equally between two persons, by a fence parallel to the perpendiculars.

Ans. The division-fence=22·8035 chains, and it divides the base into two parts, whose lengths are 17·0087 and 12·9913 chains respectively.

19. A field, in the form of an equilateral triangle, contains just half an acre; what must be the length of a tether, fixed at one of its angles, and to a horse's nose, to enable him to graze exactly half of it? *Ans.* 48·072 yards.

20. Required the diagonal of a trapezium, whose area contains 3¾

acres, and the perpendiculars on the diagonal to the opposite angles being 430 and 360 links. *Ans.* $949\frac{29}{70}$ links.

21. Required the difference between the area of the section of a round tree, 20 inches over, and its inscribed and circumscribed squares.

Ans. { 114·16 sq. in. diff. of circular section, and its inscribed square. 85·84 sq. in. diff. of its circumscribed square and circular section.

22. If from a right-angled triangle, whose base is 12, and perpendicular 16 feet, a line be drawn parallel to the perpendicular, cutting off a triangle whose area is 24 square feet; required the length of the sides of this triangle. *Ans.* 6, 8, and 10 feet.

23. Supposing the expense of paving a semicircular plot, at 2*s.* 4*d.* per square foot, come to 10*l.*, what is the diameter of it?

Ans. 14·7737 feet.

24. The four sides of a trapezium are 13, 13·4, 24, and 18 feet; and the two first contain a right angle; required the area.

Ans. 410·122 square feet.

25. The sides of three squares being 4, 5, and 6 feet, how long is the side of that square which is equal to all three?

Ans. 8·7749 feet *nearly.*

26. If the base of a triangle be 40, and the other two sides 30 and 20, what is the length of its perpendicular?

Ans. 14·52, &c.

27. If the base of a triangle be 40, and the sides 30 and 20, what are the segments of the base made by a line bisecting the vertical angle? *Ans.* 24 and 16.

28. If the radius of a circle be 10, what are the sides of the regular inscribed trigon, tetragon, pentagon, hexagon, octagon, and decagon?

Ans. 17·32, 14·142, 11·756, 10, 7·654, and 6·18, *nearly.*

29. If the side of an equilateral triangle be 10, what will be the side of another equilateral triangle, whose area is *one-fourth* of the former? *Ans.* 5.

30. If the area of a triangle be 1000, and the sides are in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, what are those sides?

Ans. 40·074, 50·093, and 66·791 *nearly.*

31. Supposing the hypothenuse of a right-angled triangle to be 40, and the difference of the other sides 10; required the sides.

Ans. 22·839 and 32·839 *nearly.*

32. If the base of a right-angled triangle be 40, and the sum of the other sides 80, what is the perpendicular? *Ans.* 30.

33. If the perpendicular of a right-angled triangle be 40, and the difference of the other sides 10; required the sides.

Ans. 75 and 85.

34. If the base and perpendicular of a right-angled triangle are each 1, what is the area of a circle having the hypothenuse for its diameter? *Ans.* 1·5708, *nearly.*

35. Required the area of an equilateral triangle, whose side is 36 feet. *Ans.* 561·18446 square feet.

36. In taking the dimensions of a trapezium, I found the first perpendicular to rise at 568, and to measure 835 links; the second at 1865, and to measure 915 links; the whole diagonal measured 2543 links; what is the area of the trapezium?

Ans. 22 *a.* 1 *r.* 0 *p.*

37. Two sides of an obtuse-angled triangle are 5 and 10 chains, what must be the length of the third side, that the triangle may contain just 2 acres of ground?

Ans. 8·06225, or 13·60147 chains.

38. The side A B of a triangular field is 40, B C=30, and C A=25 chains; required the sides of a triangle parted off by a division-fence made parallel to A B, and proceeding from a point in C A, at the distance of 9 chains from the angle A.

Ans. 16, 19·2, and 25·6 chains.

39. A field in the form of a right-angled triangle is to be divided between two persons, by a fence made from the right angle meeting the hypothenuse perpendicularly, at the distance of 880 links from one end; required the area of each person's share, the length of the division-fence being 660 links.

Ans. 2 *a.* 3 *r.* 24½ *p.*, and 1 *a.* 2 *r.* 21¼ *p.*

40. It is required to part from a triangular field, whose three sides measure 1200, 1000, and 800 links respectively, 1 *a.* 2 *r.* 16 *p.* by a line parallel to the longest side.

Ans. The sides of the remaining triangle are 927, 772½, and 618 links respectively.

41. If the triangle A B C be divided by lines drawn from C to the base A B, which measures 1125 links, into 3 parts in proportion, as 2, 3, and 5, how many links is each point of division from A ?

Ans. 225 and 562½ links.

42. The segments of the base of a triangle, made by a straight line bisecting the vertical angle, are 72 and 58, and the sum of the other two sides is 270; required the area of the triangle.

Ans. 7496·055.

43. The area of an equilateral triangle, of which the base falls on the diameter, and the vertex in the middle of the arc of a semicircle is equal to 100; required the diameter of the circle.

Ans. 26·32148.

44. The area of a plane triangle is 58800 square links, and the sum of its three sides 1120 links; required the radius of the inscribed circle. *Ans.* 105 links.

45. The continual product of the three sides of a plane triangle is 5400000, and the area of the triangle is 10800; required the diameter of the circumscribed circle. *Ans.* 250.

46. The expense of paving a semicircular plot amounted to 19*l.* 1*s.* 9½*d.*; required the diameter of the circle. *Ans.* 50 feet.

47. A triangle, of which the three sides are 140, 100, and 82, is inscribed in a circle; what is the diameter of the circle?

Ans. 142·2037.

48. A circular fish-pond occupies an acre; find the perimeter of the circumscribed square. *Ans.* 942·01753 links.

49. The perpendicular drawn from one of the angles of an equilateral triangle to the opposite side measures 12 feet; required the length of the side of the triangle. *Ans.* 13·8564 feet.

50. What difference is there between a plot of land 48 yards long and 30 yards broad, and two others each of half the dimensions?

Ans. 720 square yards.

51. The four sides of a field, whose diagonals are equal to each other, are 25, 35, 31, and 19 poles respectively; what is the area?

Ans. 4 *a.* 1 *r.* 38 *p.*

52. A maltster has a kiln, that is $16\frac{1}{2}$ feet square, he wants to pull down, and to build a new one, that will dry three times as much at a time as the old one; what must be the length of its side?

Ans. 28·578 feet.

53. How many acres are contained in a square meadow, the diagonal of which is 20 perches longer than either of its sides?

Ans. 14 *a.* 2 *r.* 11 *p.*

54. From a point within a triangular field, the sides of which were equal, I measured the distances to the three angles, and found them 12·5, 10, and 7·5 chains respectively; required the area.

Ans. 12 *a.* 1 *r.* 22 *p.*

55. It is required to lay out $4\frac{1}{2}$ acres of land in a triangular form, so that the length of one side may be 15 chains, and the lengths of the other sides in the ratio of 2 to 3; what must be the length of those sides?

Ans. 7·7914 and 11·6871 chains; or 29·58536 and 44·37804 chains.

56. In a rectangular tract of land, containing 58 *a.* 3 *r.* 8 *p*, the difference of the lengths of the sides is just equal to the difference between the lengths of the longer side and the diagonal; hence the sides are required. *Ans.* 21 and 28 chains.

57. The area of an equilateral triangle being 720, required the side. *Ans.* 40·7776.

58. A gentleman had a garden surrounded by a terrace-walk; the length of the garden is 500 yards, and the breadth 400 yards; the walk was equal to $\frac{1}{3}$ of the garden; required the breadth of the walk?

Ans. 13·6809 yards.

59. A gentleman has a garden 100 feet long, and 80 feet broad,

but intends to make a walk half round it, that shall take up half the ground; required the breadth of the walk.

Ans. 25·96876 feet.

60. If the area of an equilateral triangle inscribed in a circle be 400, what is the area of the equilateral triangle circumscribing the same circle? *Ans.* 1600.

61. A gentleman who has a triangular field which contains 5·87878 acres, and whose three sides are to each other as 5, 6, and 7, is desirous of making in it a circular plantation, the greatest that it will admit of; required its diameter and area.

Ans. 6·53197 diam. of inscribed circle, and its area = 3 *a.* 1 *r.* 16 *p.*

62. The hypothenuse of a right-angled triangle is 25 chains, and the sum of the base and perpendicular is 35 chains; required the area without finding the sides. *Ans.* = 15 acres.

63. A gentleman applied to a surveyor to measure him a field in the form of a right-angled triangle; but the base and perpendicular being surrounded with water, he could only obtain the length of the hypothenuse = 100 chains, and that of a pathway bisecting the angle at the base, and terminating in the perpendicular = 80 chains; required the area. *Ans.* 248 *a.* 0 *r.* 37½ *p.*

64. Describe a parallelogram, which shall have its area and perimeter respectively equal to the area and perimeter of a given triangle.

65. If the side of an octagonal grass-plot in a gentleman's pleasure-ground measures 100 feet, what will be the expense of making a gravel walk, from the middle of one of its sides to the middle of the opposite side, at 3½*d.* per yard lineal measure?

Ans. 80·4737 yards, the length of the walk, and 1*l.* 3*s.* 5½*d.* the expense.

66. Suppose a ladder 100 feet long, placed against a perpendicular wall 100 feet high, how far would the top of the ladder move down the wall, by pulling out the bottom thereof 10 feet?

Ans. 6·0150756 inches.

67. Suppose a person whose height is 5ft. 7in. travels 10000 miles in the arc of a great circle; how much will his head have gone farther than his feet, the circumference of the earth being 21600 miles?

Ans. = 16·2412 feet the excess.

68. Three persons, A, E, and Y, bought a piece of land in the form of an ellipse for 180*l.* The conjugate axis is 40 chains, and the distance of the focus from the centre 15 chains, the abscissa of Y's part is to be 18 chains, and the rectangular ordinate of E's part 15 chains; A's share lies between those of Y and E, what is each person's share of the land, and what sum ought each to pay?

Ans.

Sq. chains.		£. s. d. q.
175·99172 = E's part;	and	20 . 3 . 4 . 0·512 E's money.
509·10110 = Y's part;	and	58 . 6 . 9 . 1·152 Y's money.
885·70350 = A's part;	and	101 . 9 . 10 . 2·336 A's money.

69. Divide a common of 739 acres, 1 rood, among A, B, C, and D, in proportion to their rents, which are,—A's, 200*l.*; B's, 133*l.* 6*s.* 8*d.*; C's, 113*l.* 7*s.* 9*d.*; and D's, 78*l.* 11*s.* 2*d.* a year.

Ans.
A's, 281 *a.* 1 *r.* 35·10, &c. *p.*
B's, 187 *a.* 2 *r.* 23·40, &c. *p.*
C's, 159 *a.* 2 *r.* 12·08, &c. *p.*
D's, 110 *a.* 2 *r.* 9·41, &c. *p.*

70. Divide a common, of 244 acres, 3 roods, 30 poles, among A, B, C, and D, whose estates, on which their claims are founded, are respectively 500*l.*, 450*l.*, 150*l.*, and 100*l.* a year; the quality of the land allotted to each claimant being of the respective values of 20*s.* 18*s.* 15*s.* and 12*s.* an acre.

Ans.
A = 89 *a.* 2 *r.* 17·80 *p.*
B = 89 *a.* 2 *r.* 17·80 *p.*
C = 35 *a.* 3 *r.* 15 12 *p.*
D = 29 *a.* 3 *r.* 19·27 *p.*

71. A common consists of 5 *a.* 1 *r.* 24 *p.*, valued at 16*s.* an acre; 6 *a.* 1 *r.* 8 *p.*, at 14*s.*; and 6 *a.* 3 *r.*, at 12*s.*; how must it be divided

among A, B, C, and D, whose farms, on which their claims are founded, are respectively 200*l.*, 150*l.*, 80*l.*, and 30*l.* a year?

Ans.

	Quantities. *a.* *r.* *p.*	Values. *£.* *s.* *d.*	
A's share =	5 . 1 . 24 at 16..	= 4 . 6 . $4\frac{3}{4}$	A's.
	1 . 3 . $2\frac{1}{2}$ at 14*s.*	= 1 . 4 . $8\frac{1}{2}$	
B's share =	4 . 2 . $5\frac{3}{4}$ at 14*s.*	= 3 . 3 . 6	B's.
	1 . 2 . $24\frac{1}{2}$ at 12*s.*	= 19 . 10	
C's share =	3 . 2 . 33 at 12*s.*	= 2 . 4 . $5\frac{3}{4}$	C's.
D's share =	1 . 1 . $22\frac{1}{2}$ at 12*s.*	= 16 . 8	D's.

72. An irregular piece of ground, consisting of 420 acres, 3 roods, 14 perches, is to be exchanged for a square piece of the same surface; what will be the length of one of its sides? This square is likewise to be divided into 40 equal shares; what will be the extent of a side of each?

Ans.
- 1427·1837 yds., the length of one of the sides of the square field.
- 225·6575 yds., the side of each of the parts into which the whole field is divided.

73. Three farmers, A, B, and C, had each an equal share of a triangular field, whose base, being a river, measured half a mile, its content being 120 acres; the marshes were drawn parallel to the base. Now C had his share next the river, B the middle division, and A the rest. They agreed with a contractor to dig a ditch from the top of A's division, perpendicular to the river-side. The breadth of the ditch, at a medium, is 6 feet, and depth $4\frac{1}{2}$ feet. Required the contractor's charge against each of the three farmers, at 4*d.* per solid yard.

Ans. =
- A pays 38*l.* 2*s.* $2\frac{1}{2}$*d.*
- B pays 15*l.* 15*s.* $7\frac{1}{2}$*d.*
- C pays 12*l.* 2*s.* 2*d.*

74. Suppose an estate of 575 acres, 2 roods, 4 perches, is to be let to two farmers, the one to have 100 acres more than the other? how much land has each? And what is the annual rent which each far-

mer has to pay, supposing the rent to be 3*l.* per acre, for 90 acres in each person's farm, and 2*l.* 10*s.* per acre for the remainder?

Ans. =

Content of each farm.	Annual rents.
337 *a.* 3 *r.* 2 *p*.....	889*l.* 8*s.* 8½*d.*
237 *a.* 3 *r.* 2 *p*.....	629*l.* 8*s.* 1½*d.*

75. A landed squire two daughters had,
And both were very fair;
He gave to each a piece of land,
One round, the other square.
At twenty pounds an acre just,
Each piece its value had;
The shillings that encompass each
The price exactly paid.
If 'cross the shilling be an inch,
(And it is very near,)
How much above the circle is
The excess of the square?

ACRES.	£
627·264 area of square, and	12545·28 value of square.
492·653 area of circle, and	9853·063 value of circle.
diff. = 134·611 excess of square, and	2692·217 = 2692*l.* 4*s.* 4*d.*

Consequently, the square is the better portion, by 134·611 acres, or 2692*l.* 4*s.* 4*d.* = *Ans.*

THE END.

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